Additive Manufacturing – Module 5

Spring 2015

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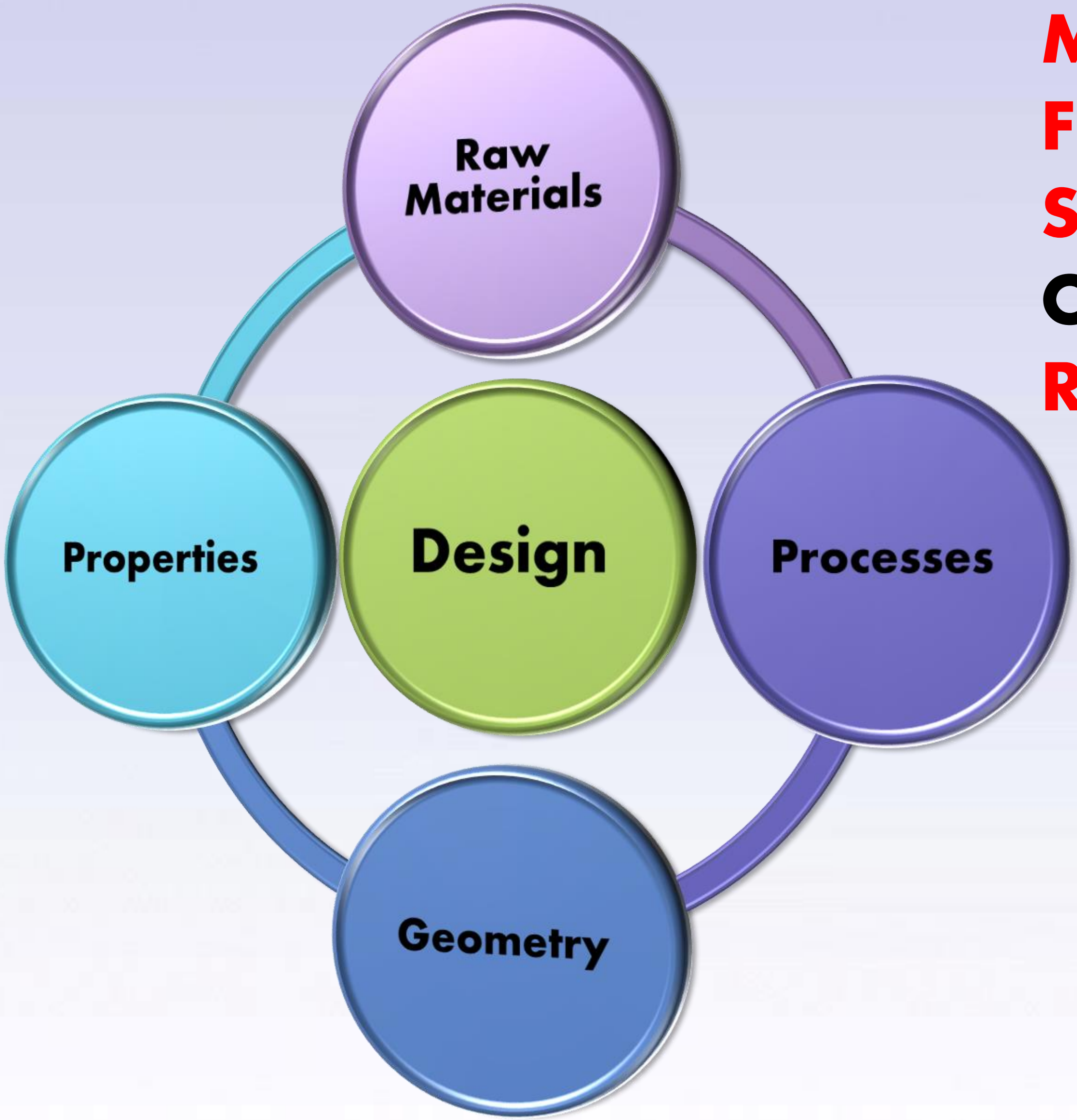
The Department of Mechanical Engineering
University of Arkansas, Fayetteville

Design for AM

- Introduction
- 3D Scanning
- CAD
  - Curve
  - Surface
  - Solid



Make Functional Stuff Out of Raw Materials

Design for AM

Introduction
3D Scanning
CAD
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Solid

Geometry
- 3D scan
- CAD
- Complex geometry

Material
- Material composition
- Multiscale structure
- Smart material (4D)

Process
- Process parameter in time
- Process parameter in space

Property
- CAE

Geometry

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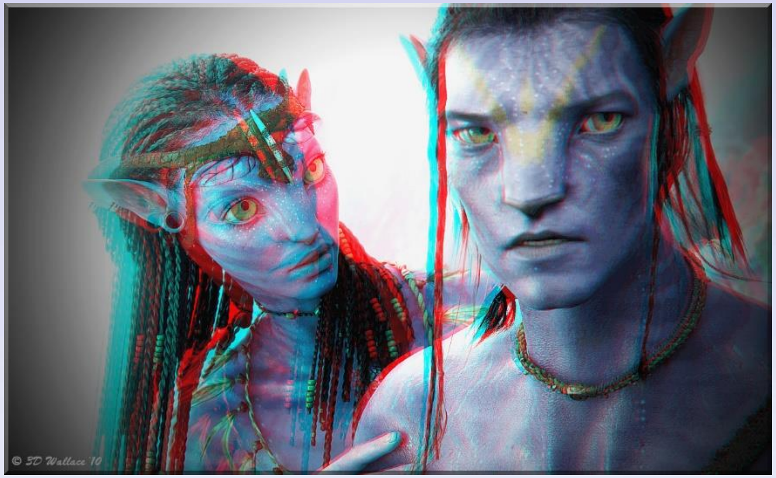
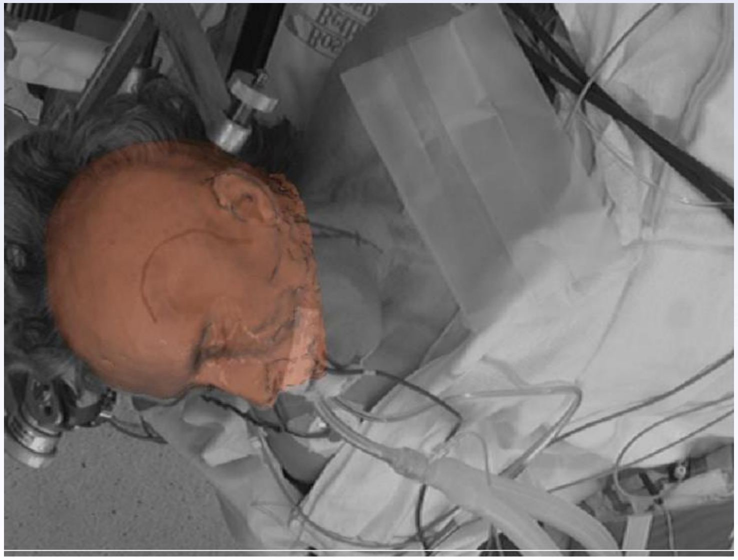
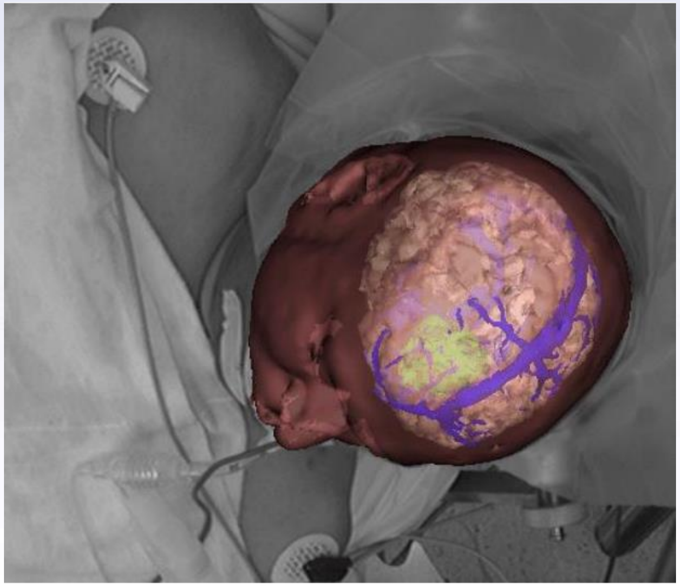
3D scanning



3D image

Vision – has depth info





Medical applications

Picture Credit: Dr. Wojciech Matusik@MIT

Geometry

Advanced Manufacturing | Modeling | Materials

Introduction

3D Scanning

CAD

Curve

Surface

Solid

3D scanning

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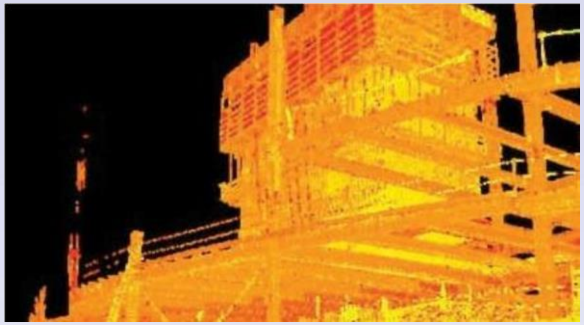
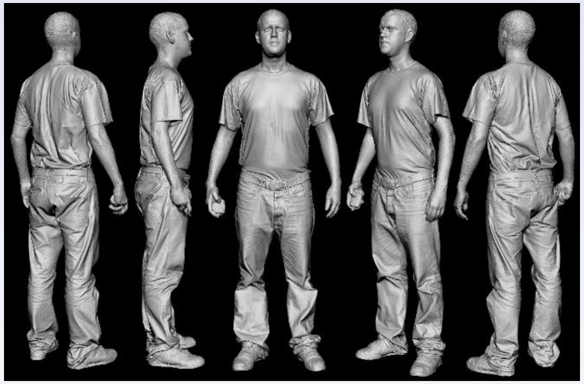


Building scan for quality control

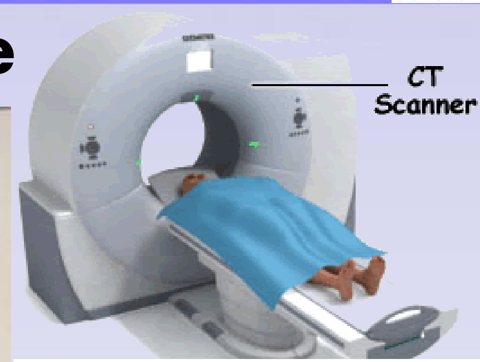


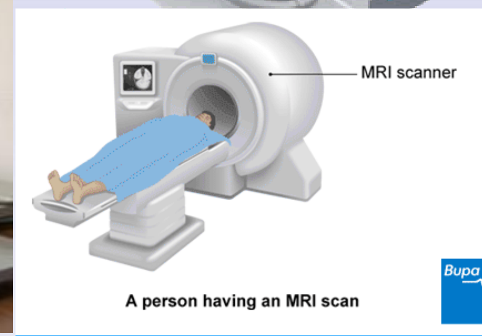
Art, fashion, person, etc.

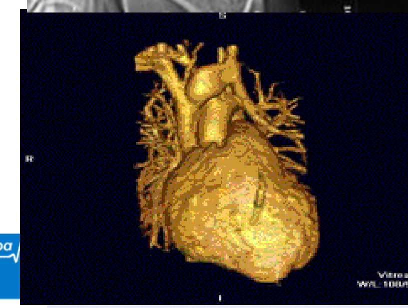
3D content for 3D printer, replication

Geometry

- 3D scanning – range

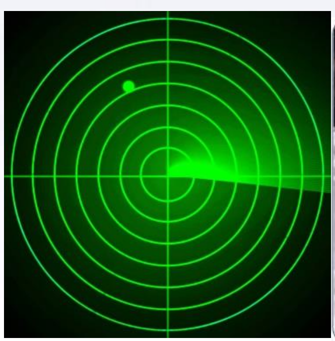


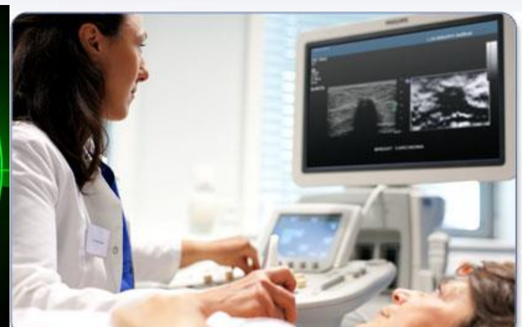


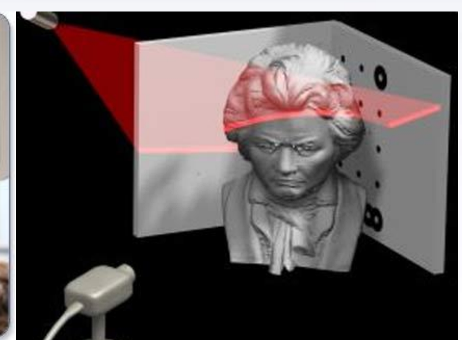


Contact scanner
- Mechanical (CMM)
- Accurate but slow (<~100Hz)

Non-Contact Transmissive Scanner
- CT
- MRI







Non-contact Optical scanner
- Radar
- Sonar
- Optical

Geometry

- 3D scanning – Optical scan

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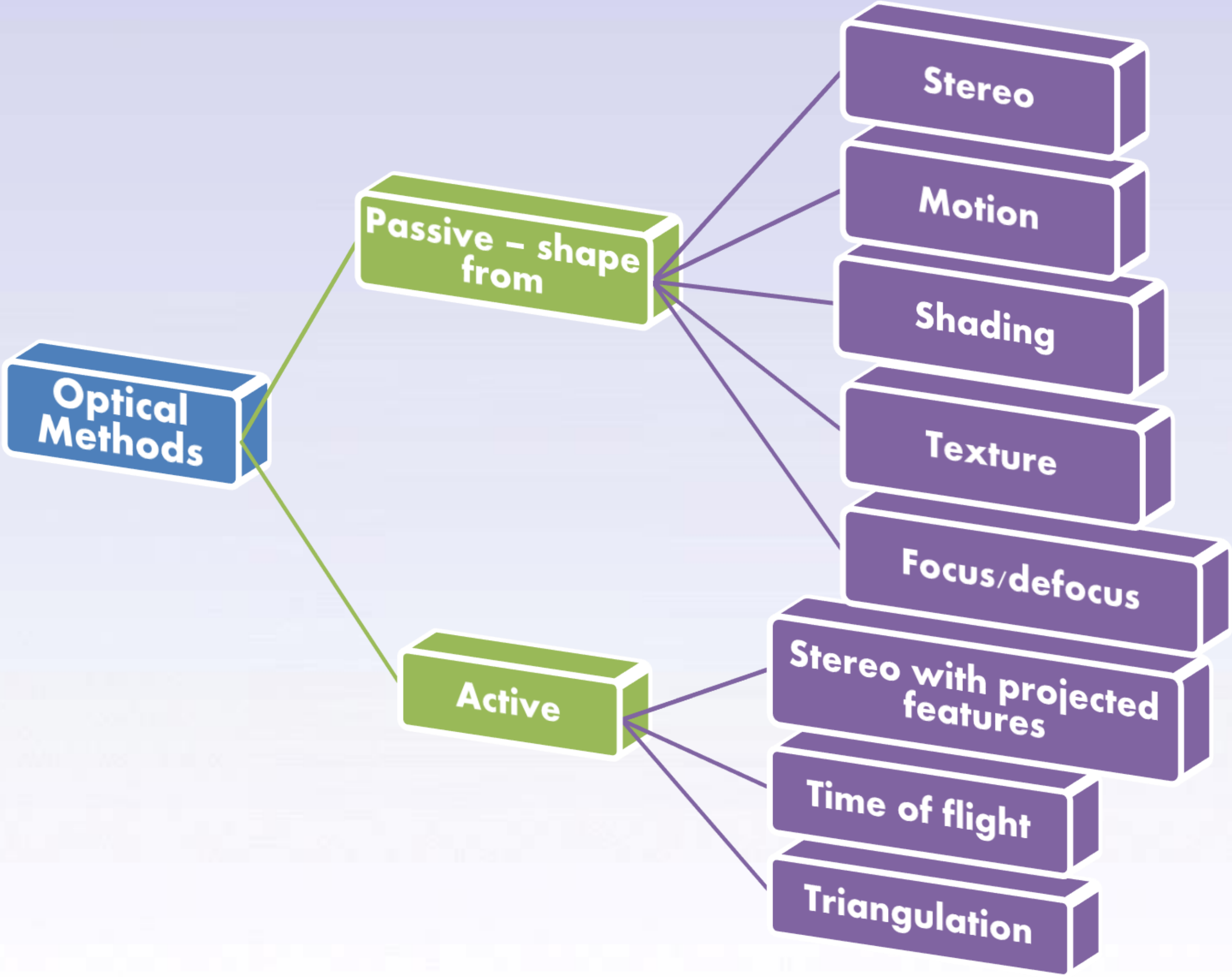

Geometry

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3D scanning – Optical scan

Pros

- Non-contact
- Safe
- Usually inexpensive
- Usually fast

Cons

- Sensitive to transparency
- Often confused by specularity & inter-reflection
- Texture (sometimes helpful, sometime not)

Slide Credit: Dr. Wojciech Matusik@MIT

Geometry

Introduction

3D Scanning

CAD

Curve

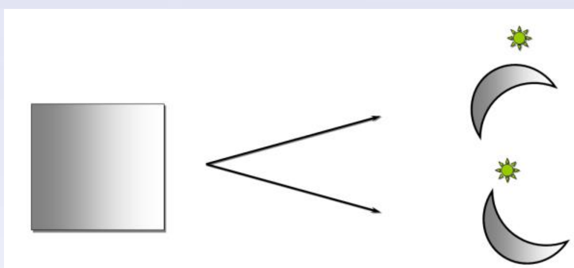
Surface

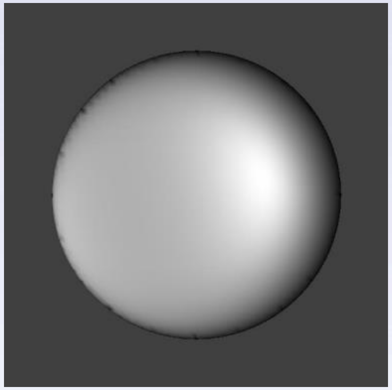
Solid

3D scanning – Optical scan – passive

Shape from shading

- Given: image of surface with known, constant reflectance under known point light
- Estimate normals, integrate to find surface
- Problem: ambiguity





Pros

- Single image
- No correspondences

Cons

- Mathematically unstable
- Can't have texture

Slide Credit: Dr. Wojciech Matusik@MIT

Geometry

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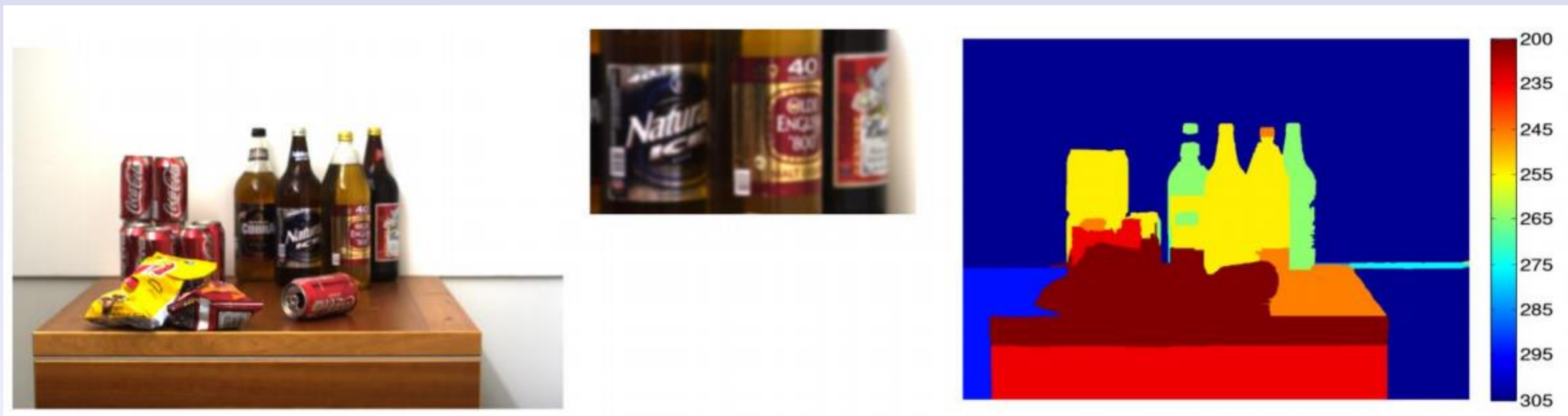
3D scanning – Optical scan – passive

Shape from focus/defocus

- at which focus setting is a given image region sharpest
- how out-of-focus is each image region



Pros

- Single image
- No correspondences

Cons

- Mathematically unstable
- Inaccurate

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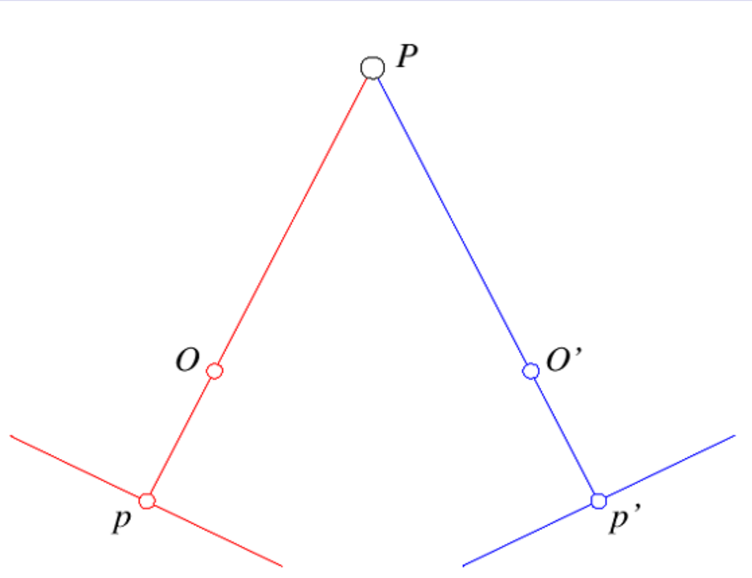
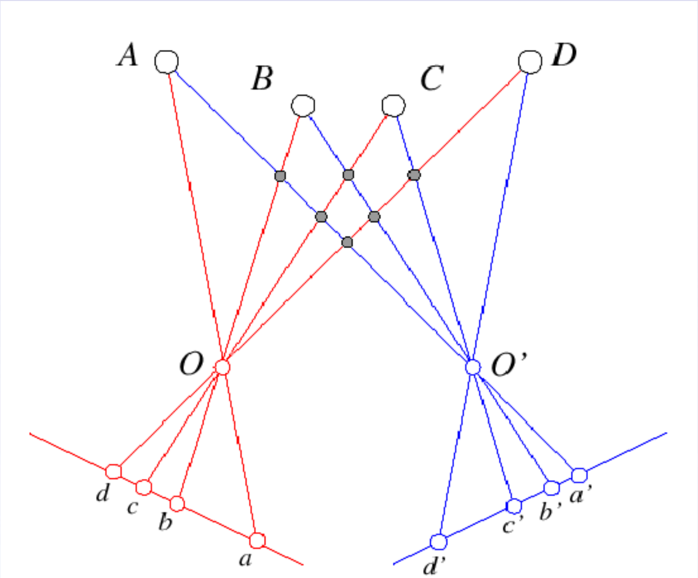
Geometry

3D scanning – Optical scan – passive

Stereo

- Two cameras – like eyes





Pros

- Passive
- Cheap hardware (2 cameras)
- Easy to accommodate motion
- Intuitive (similar to human vision)

Cons

- Need features for correspondence
- Noisy data (inaccurate)
- Bad around silhouettes

Slide Credit: Dr. Wojciech Matusik@MIT

Geometry

Introduction

3D Scanning

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Curve

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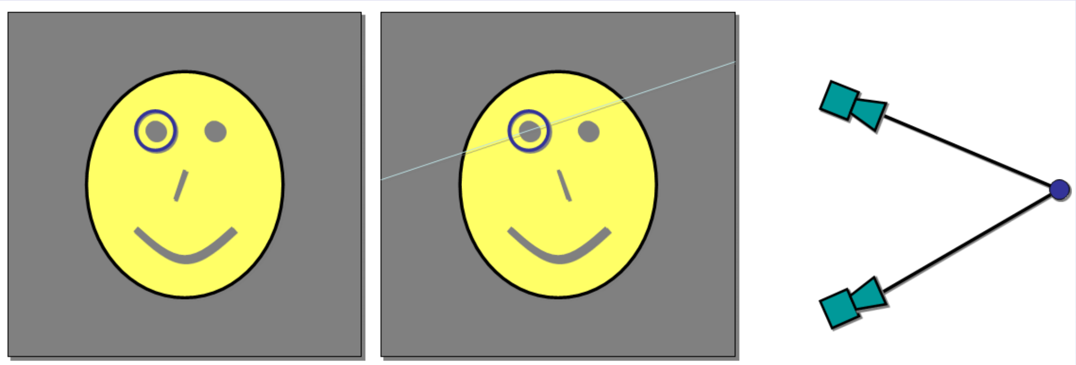
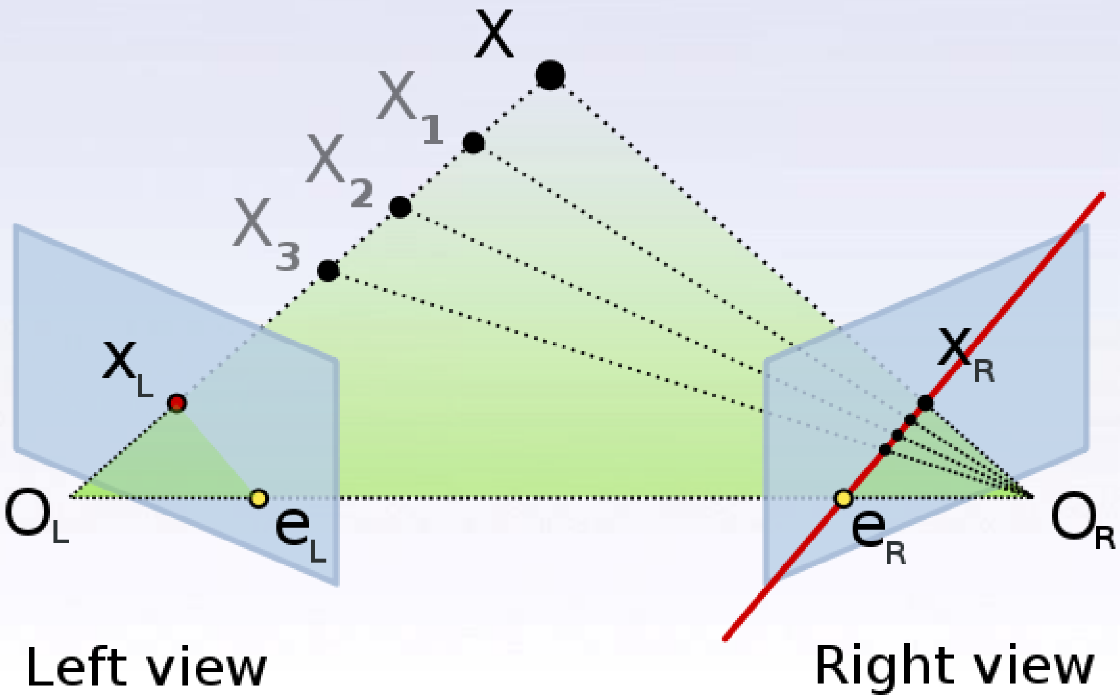
Solid

3D scanning – Optical scan – passive

Stereo

Picture Credit: Dr. Wojciech Matusik@MIT





Geometry

- 3D scanning – Optical scan – active

Pros

- Can get dense data
- Much more robust and accurate than passive

Cons

- Introduce light into scene (distracting, etc.)
- More expensive

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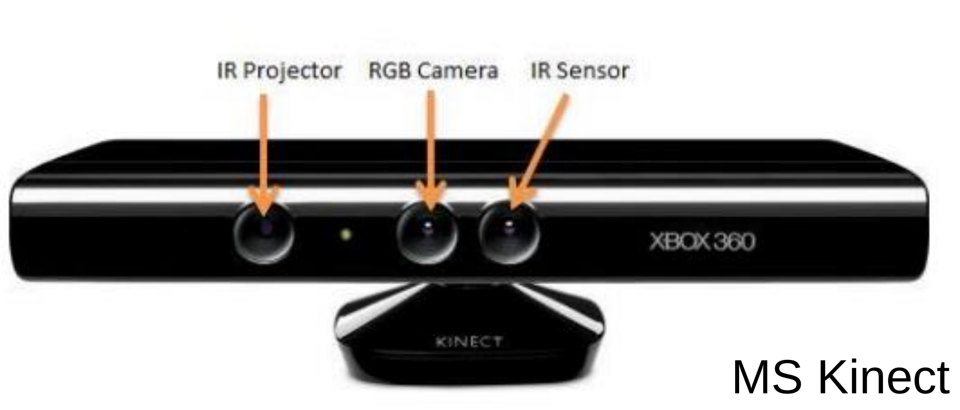
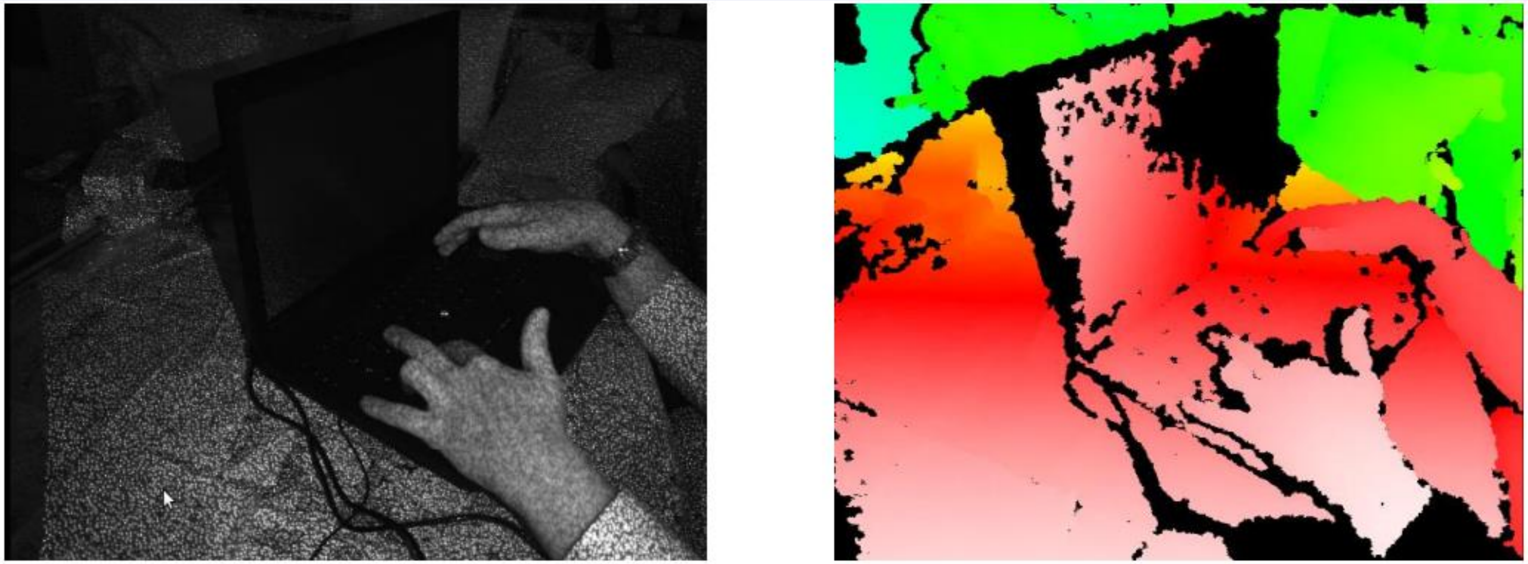
Geometry

3D scanning – Optical scan – active

Stereo with projected texture



MS Kinect



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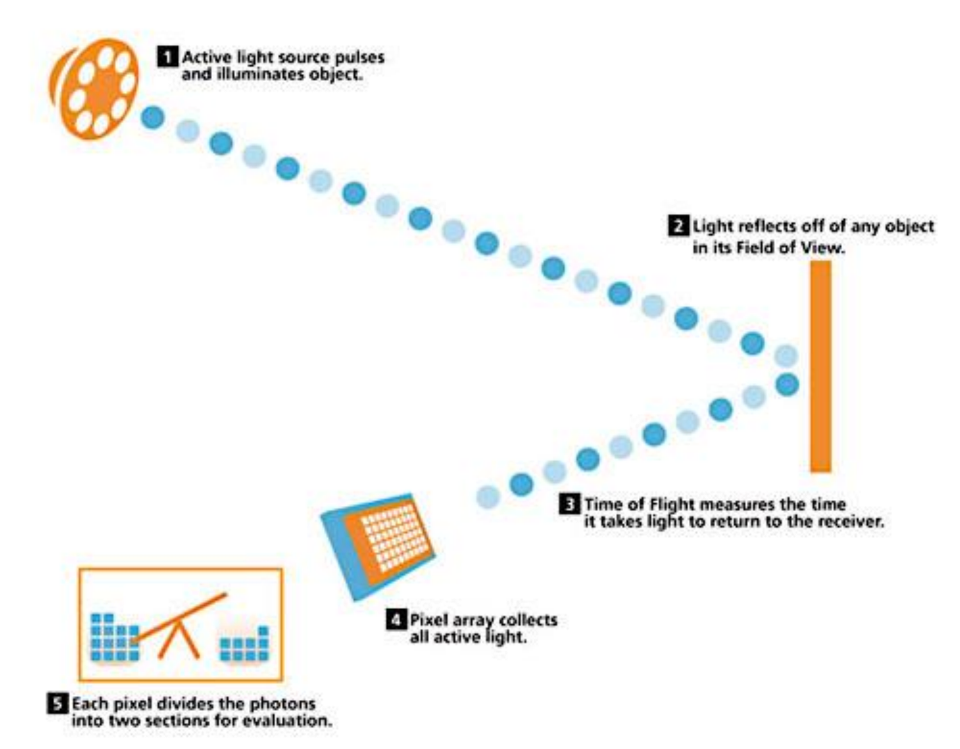
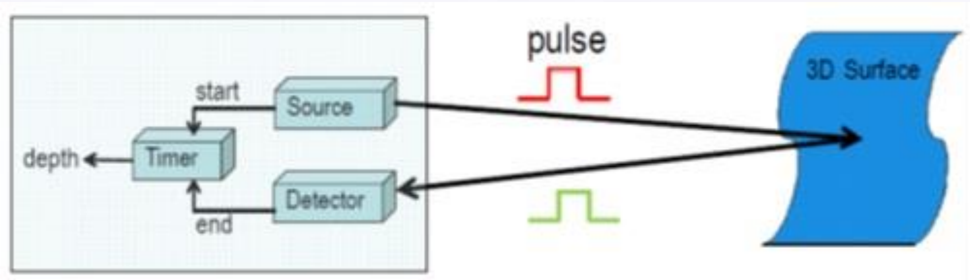
Geometry

3D scanning – Optical scan – active

Pulsed time of flight

- Send out pulses of light and time how long to return





Pros

- Large working volume

Cons

- Inaccurate (at best ~5 mm)
  - Need timing < 30 ps

- Typically used for scanning buildings, archeological sites

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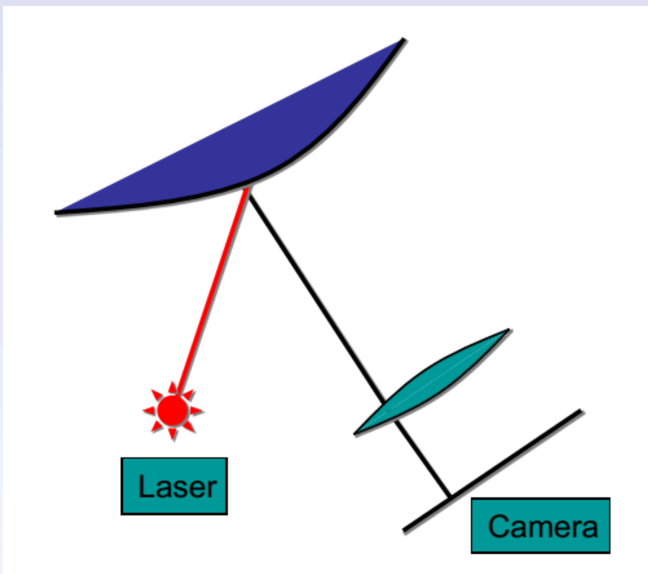
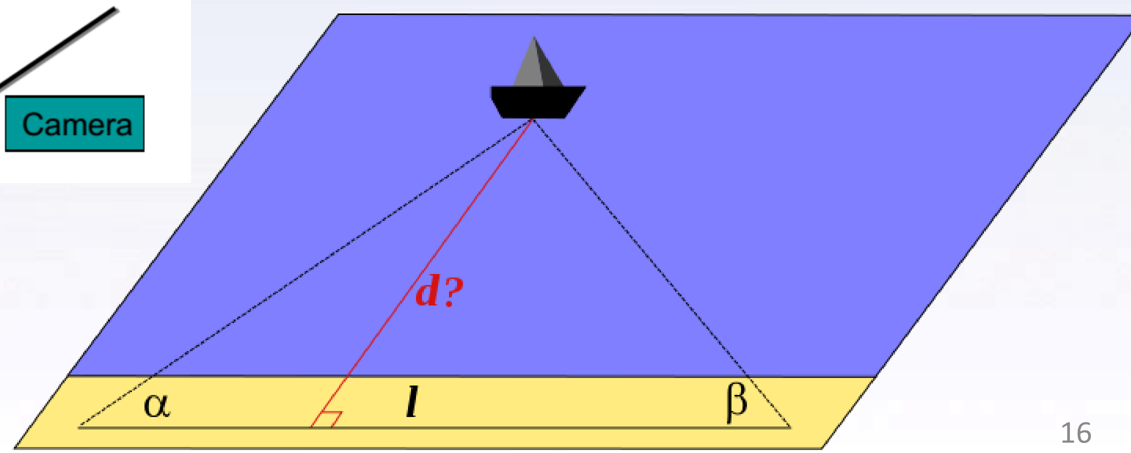
- 3D scanning – Optical scan – active

Triangulation

- process of determining the location of a point by measuring angles to it from known points at either end of a fixed baseline





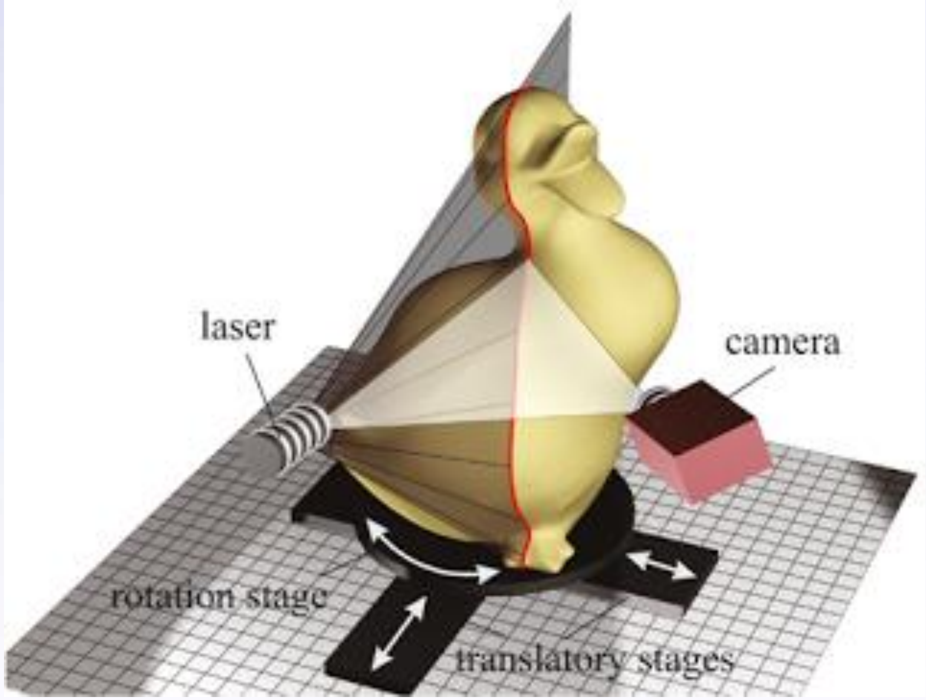
Slide Credit: Dr. Wojciech Matusik@MIT

Geometry

- **3D scanning – Optical scan – active**

Triangulation – Light strip scanning

- **Project a light stripe of laser light**
- **Scan across surface**
- **Very precise, but need many images**



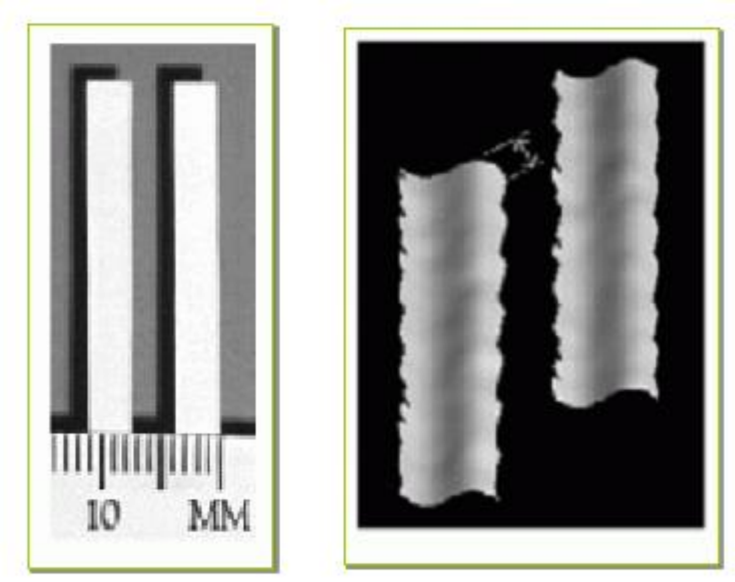
Multi-stripe laser triangulation

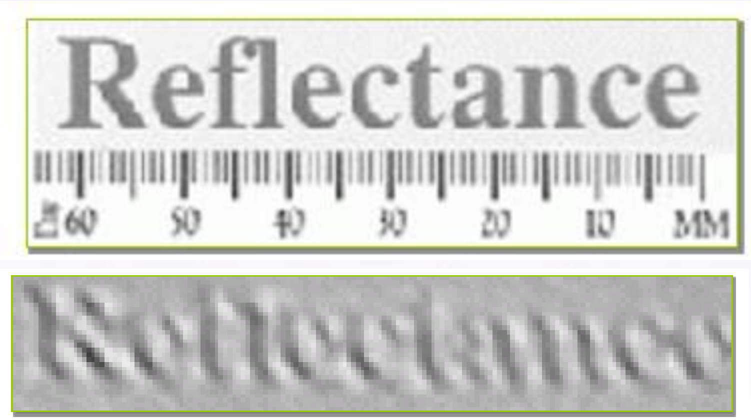
Geometry

3D scanning – Optical scan – active

Triangulation scanner issues

- Accuracy proportional to working volume (typically 1000:1)
- Can scale down to small working volume, but doesn't scale up
- Shadowing issue
- Triangulation angle: non-uniform resolution (useful angle range 15° to 30°)
- Material properties (dark, specular, etc)
- Subsurface scattering
- Laser speckle
- Edge curl
- Texture embossing





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Geometry

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3D scanning – Optical scan – active

Triangulation – Multi-stripe

- Go faster;
- Need to determine which strip is which: color or time-coded

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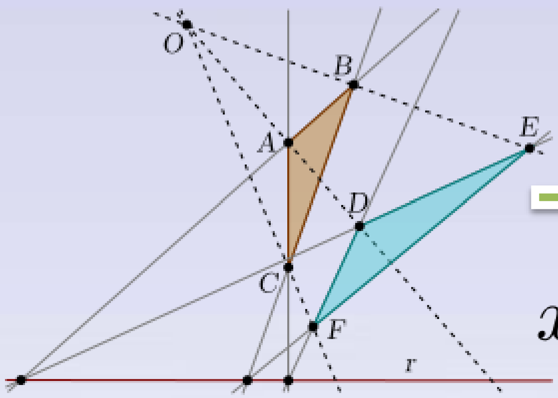
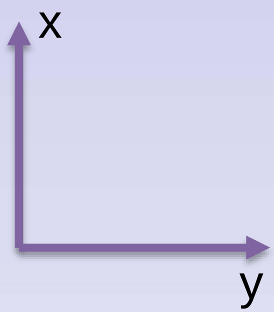
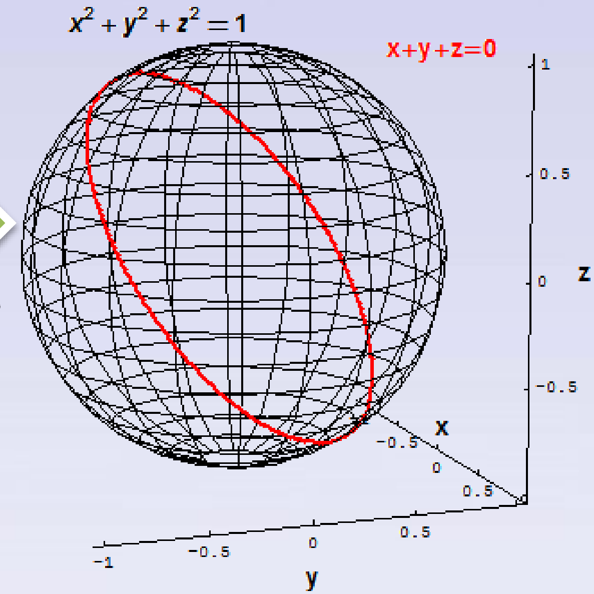
Geometry

CAD





$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Geometry

- **Shape, size, position**
- **Properties of space**

- **Coordinate system**
- **Algebra**

- **Algebraic geometry**

Geometry

CAD – Curves (1D)

$y = f(x) = ax + b;$

Explicit

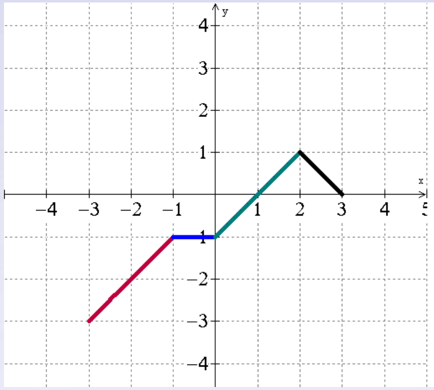
$f(x,y) = x^2+y^2=1;$

Implicit

$x(t) = \sin(t);$
$y(t) = \cos(t)$

Parametric

- Shape independent of position
- Invariant under rotation or translation
- Enable user control



Piecewise

- Discrete
- Flexible
- Each piece can be explicit, implicit, or parametric
- continuity (0^{th} order, 1^{st} order, 2^{nd} order, etc.)

Geometry

CAD – Curves (1D) – Parametric

Cubic curve – polynomial

$x(u) = a_{3x}u^3 + a_{2x}u^2 + a_{1x}u + a_{0x}$

$y(u) = a_{3y}u^3 + a_{2y}u^2 + a_{1y}u + a_{0y}$

$z(u) = a_{3z}u^3 + a_{2z}u^2 + a_{1z}u + a_{0z}$

Derivative

$x^u(u) = dx(u) / du$

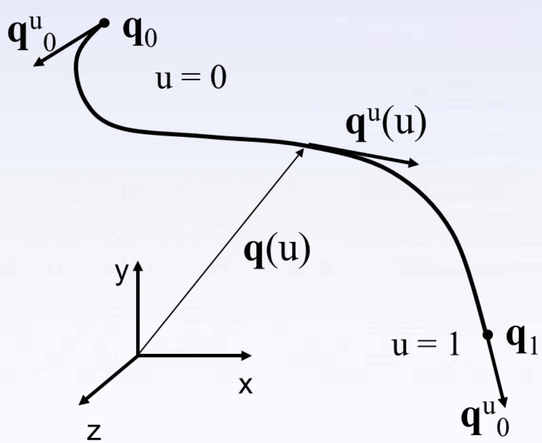
$y^u(u) = dy(u) / du$

$z^u(u) = dz(u) / du$

Cubic curve – vector form

$$\mathbf{p}(u) = \mathbf{a}_3u^3 + \mathbf{a}_2u^2 + \mathbf{a}_1u + \mathbf{a}_0$$
$$= [u^3\ u^2\ u\ 1][\mathbf{a}_3\ \mathbf{a}_2\ \mathbf{a}_1\ \mathbf{a}_0]^T = \mathbf{U\ A}$$

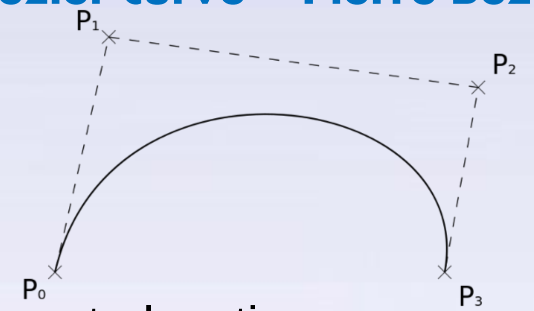
Hermite curve



end points: p(0), p(1)

end tangents: $p^u(0)$, $p^u(1)$

Bezier curve – Pierre Bezier in 1960s



4 control vertices: p_0, p_1, p_2, p_3

$$p(u) = (1 - u)^3 p_0 + 3(1 - u)^2 u p_1 + 3(1 - u)u^2 p_2 + u^3 p_3$$

- **Characteristic Polygon: Curves interpolate first and last CV.**
- **Intermediate CV's shape the curve.**
- **Changes tend to be localized.**
- **Curve is tangent to first pair of CV's and last pair of CV's.**
- **Invariant under rotations and translations.**

Geometry

CAD – Surfaces (2D)

$z(x,y) = 3x^2 + 4y^2 + 2xy$

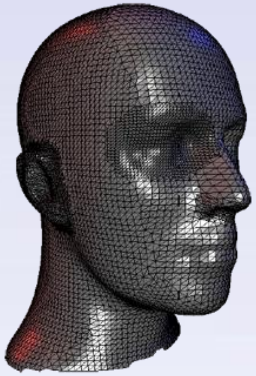
Explicit

$f(x,y,z) = x^2+y^2+z^2= 1$

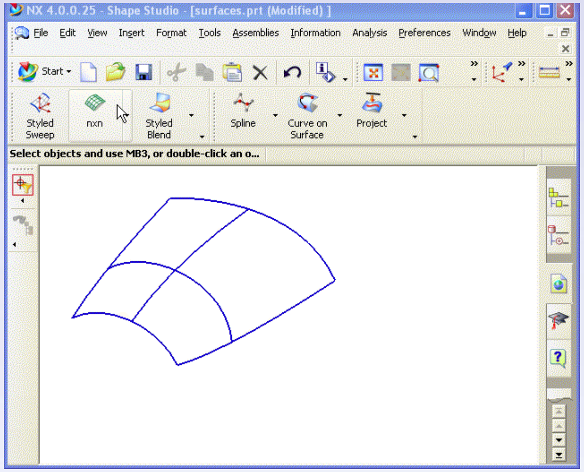
Implicit

$x=x(u,w);$
$y=y(u,w);$
$z=z(u,w);$

Parametric



Piecewise (surface patches): consider continuity



Surface lofting (create surface from curves)

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Geometry

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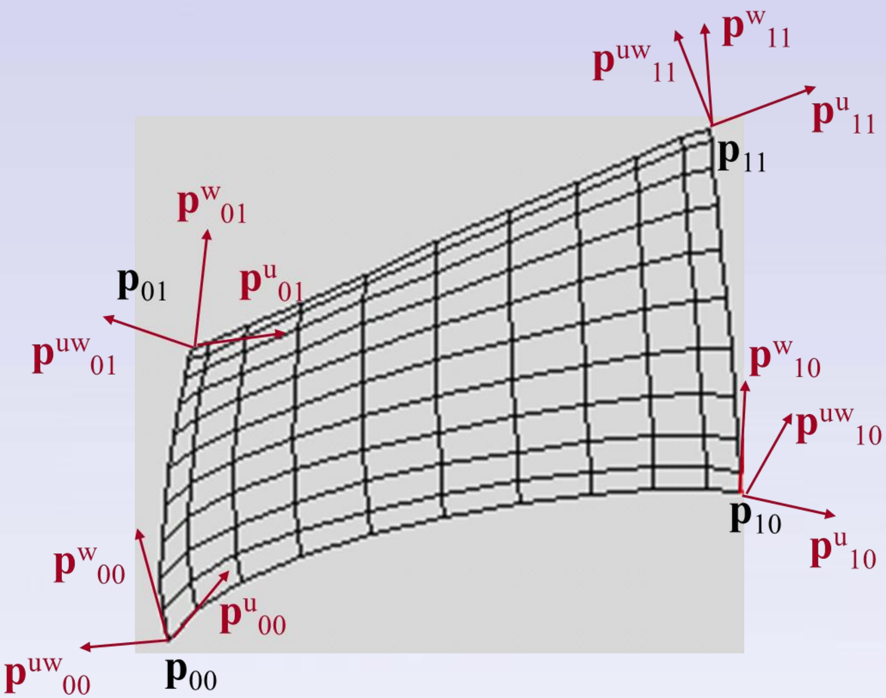
CAD – Surfaces (2D) – Parametric



Hermite form

- 4 End Points: p(0,0), p(1,0), p(0,1), p(1,1)
- 8 Tangents at end points, pu(u,w), pw(u,w)
- 4 Twist Vectors at end points: puw(u,w)

$$\mathbf{p}(u,w) = \sum_i \sum_j a_{ij} x^i y^j$$

$$\mathbf{p}(u,w) = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} a_{ij} \end{bmatrix} \begin{bmatrix} 1 \\ w \\ w^2 \\ w^3 \end{bmatrix}$$

$$\mathbf{p}(u,w) = \mathbf{UAW}$$

Slide Credit: Dr. David Rosen@GaTech

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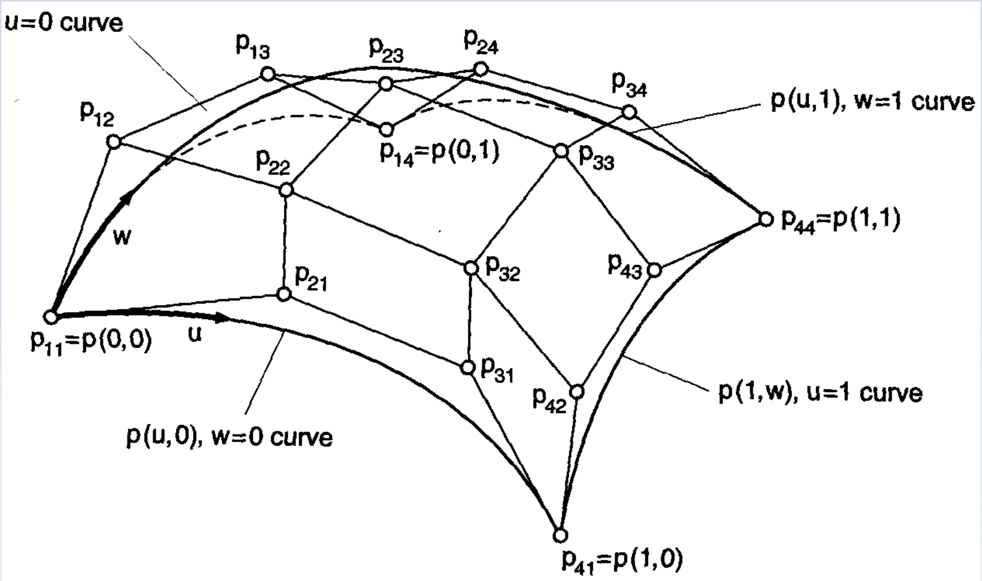
CAD – Surfaces (2D) – Parametric



Bezier surface

- 4 corner points: p00, p03, p30, p33.
- 4 intermediate points: p11, p22, p12, p21 control cross slopes in same manner as twist vectors.
- 8 other points control boundary curves.

$$\mathbf{p}(u,w) = [(1-u)^3 \quad 3u(1-u)^2 \quad 3u^2(1-u) \quad u^3]\,\mathbf{P}\begin{bmatrix}(1-w)^3 \\ 3w(1-w)^2 \\ 3w^2(1-w) \\ w^3\end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} \\ p_{10} & p_{11} & p_{12} & p_{13} \\ p_{20} & p_{21} & p_{22} & p_{23} \\ p_{30} & p_{31} & p_{32} & p_{33} \end{bmatrix}$$

Slide Credit: Dr. David Rosen@GaTech

Geometry

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Introduction

3D Scanning

CAD

- Curve
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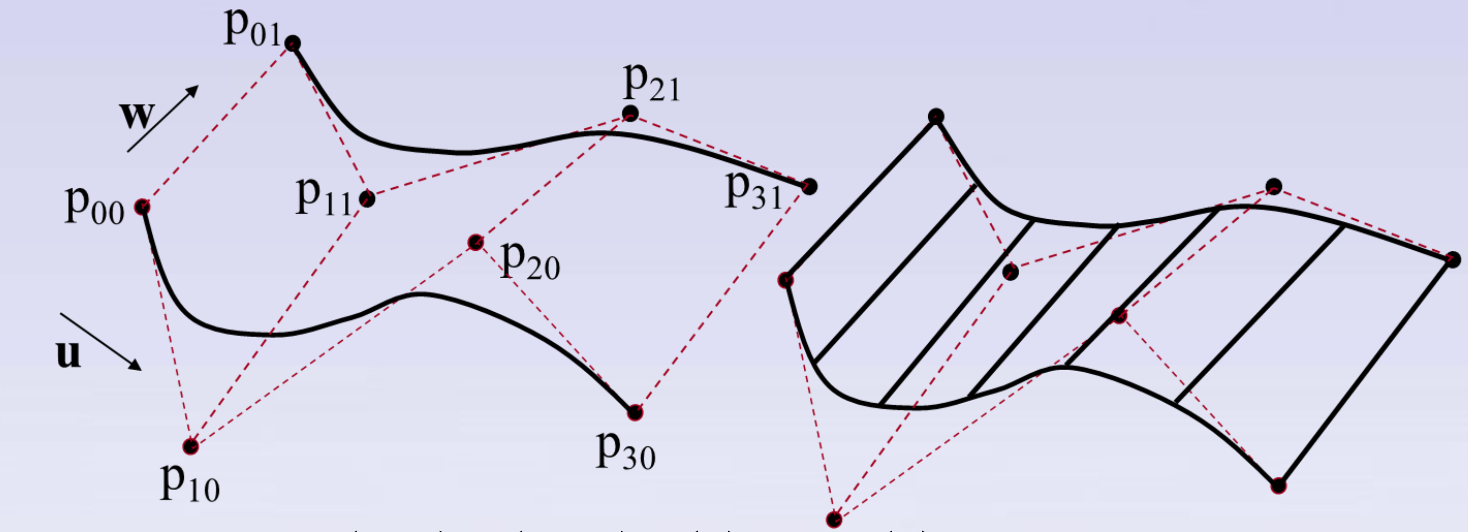
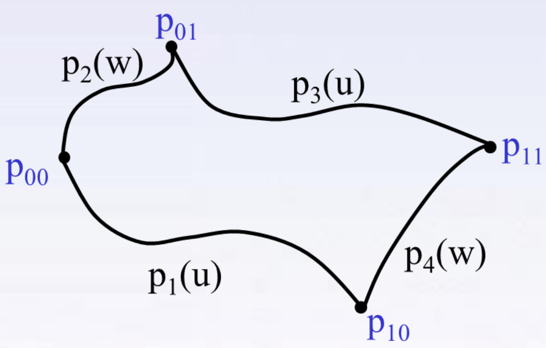
CAD – Surfaces (2D) – Loft



Linear Loft $\mathbf{p}(u,w) = (1-w)\mathbf{p}_1(u) + w\mathbf{p}_2(u)$

Bilinear Loft



Linear loft from $p_1(u)$ to $p_3(u)$

$$\mathbf{p}(u,w) = (1-w)\mathbf{p}_1(u) + w\mathbf{p}_3(u)$$

Linear loft from $p_2(u)$ to $p_4(u)$

$$\mathbf{p}(u,w) = (1-u)\mathbf{p}_2(w) + u\mathbf{p}_4(w)$$

Combine

$$\mathbf{q}(u,w) = (1-w)\mathbf{p}_1(u,0) + w\mathbf{p}_3(u,1) + (1-u)\mathbf{p}_2(0,w) + u\mathbf{p}_4(1,w)$$
$$-\mathbf{p}_{00}(1-u)(1-w) - \mathbf{p}_{01}(1-u)w - \mathbf{p}_{10}u(1-w) - \mathbf{p}_{11}uw$$

Slide Credit: Dr. David Rosen@GaTech

Geometry

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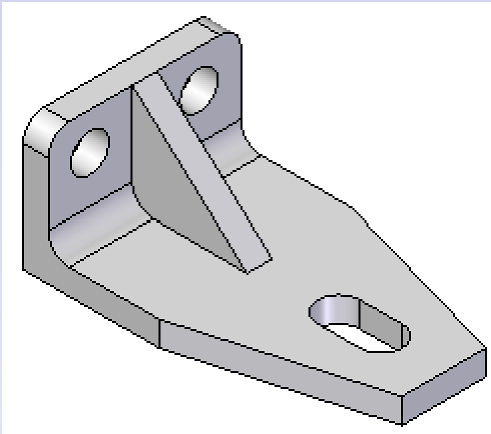
CAD – Solid(3D)



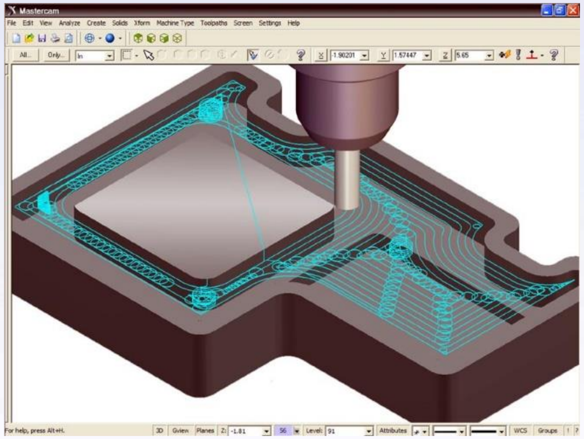
Solid model

- 3D point sets
- Inside & outside
- Physical properties: mass, volume, moment of inertia, stress, strain, etc.

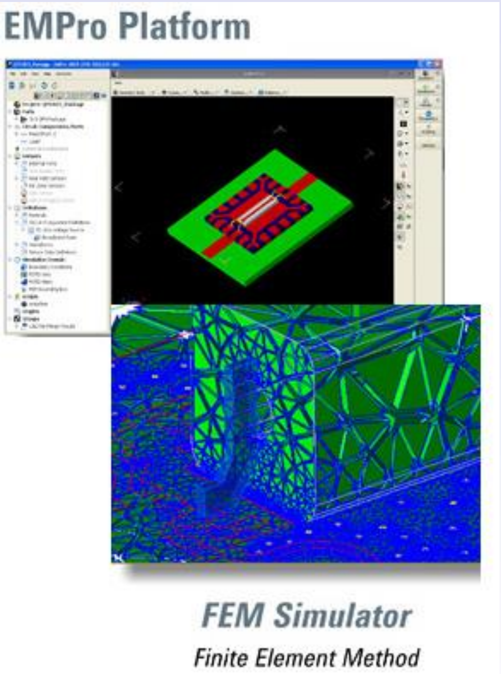
Emphasis on physical fidelity

Motivation

CT scan (generate solid data)



CAD/CAM need solid info



Physically based simulations (e.g., FEM)

Pictures from Google Image

Geometry

- CAD – Solid(3D)

General requirements

- Expressive power: have adequate info to answer any geometric questions
- Validity: manufacturability and realizability
- Unambiguity: unique representation
- Easy for transformation and Boolean operations
- Conciseness: storage requirement
- Computational ease: easy to write algorithm for
- Efficient display

General approaches

- Edges: wireframe model
- Surfaces: surface boundary
- Volume: entire volume

Geometry

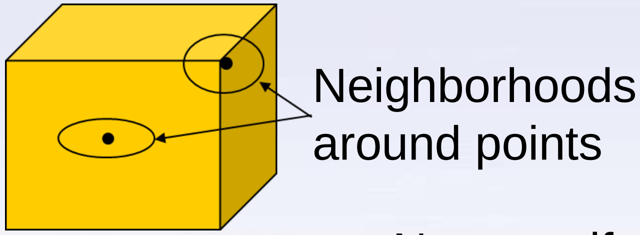
CAD – Solid(3D)

Validity of 3D models

- **Definition: A solid is a bounded, closed subset of E^3 (Euclidean Space)**
- **Bounded: finite extent; Closed: has a boundary**

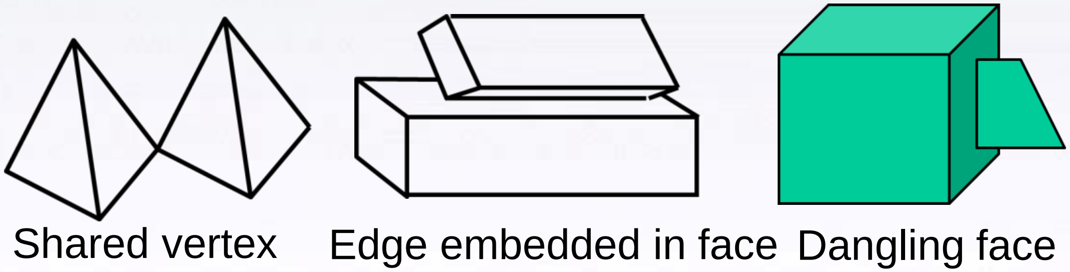
2-manifold

- **A 2-manifold M is a topological space where every point has a neighborhood topologically equivalent to an open disk of E^2.**



No isolated/embedded points or line segments

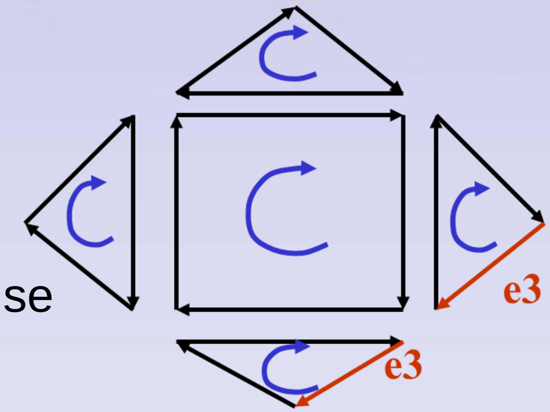
Non-manifold



Pictures credit: Dr David Rosen@GaTech

Geometry

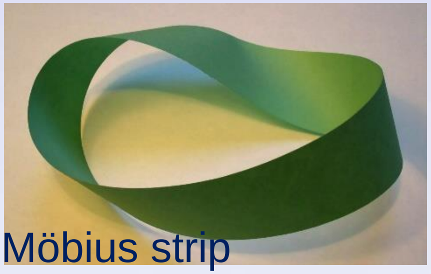
CAD – Solid(3D)



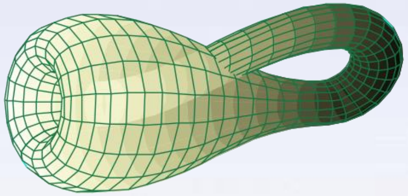
Not all 2-manifold are realizable

- **Sufficient condition: orientability**

Orient a pyramid: all polygons oriented clockwise as seen from outside (consistently oriented).



Möbius strip



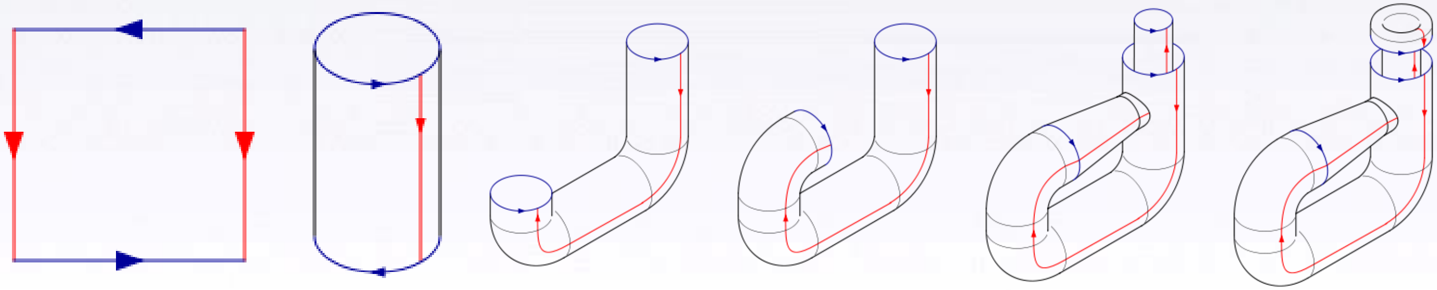
Klein bottle

Euler Characteristic

- **v - e + f = 2(s - h); s = # shells, h = # holes**

v = 5, f = 5, e = 8
v - e + f = +2
2(s - h) = +2

+2 = Euler Characteristic for all objects topologically equivalent to sphere



Pictures source:wikipedia

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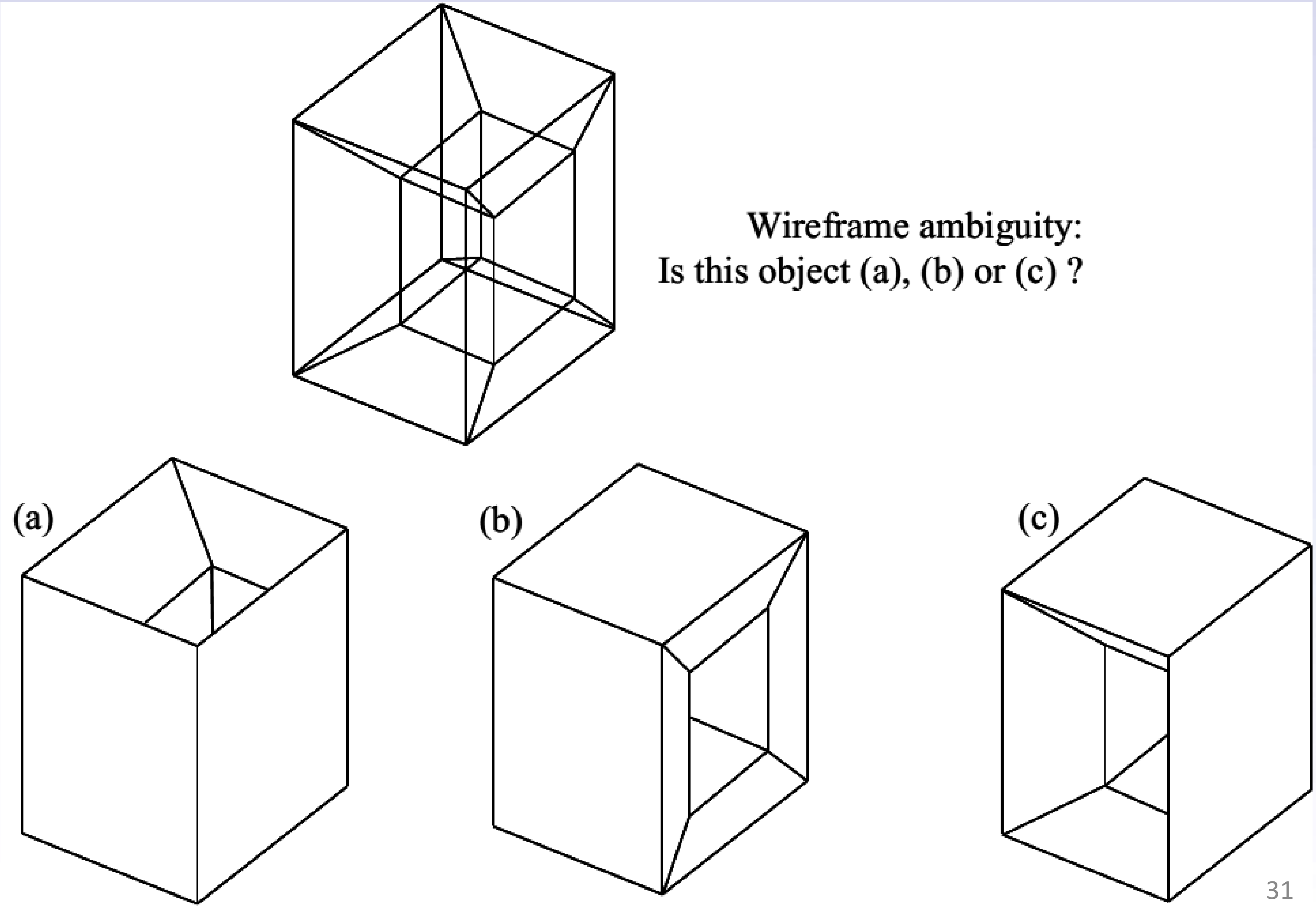
CAD – Solid(3D)

Wireframe modeling – Problem: ambiguity



CAD – Solid(3D) – Implicit representation

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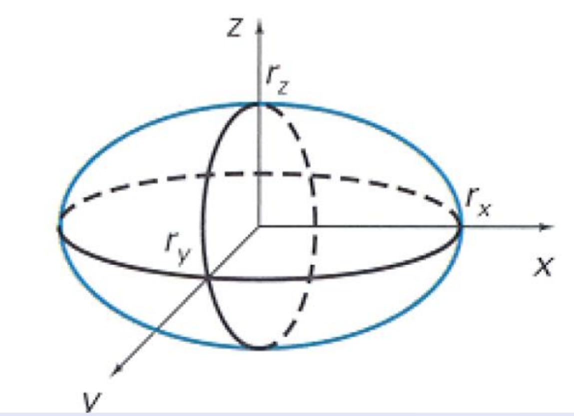
Common quadratic shapes

- Sphere
- Ellipsoid →
- Torus
- Paraboloid
- Hyperboloid

$$\left(\frac{x}{r_x}\right)^2+\left(\frac{y}{r_y}\right)^2+\left(\frac{z}{r_z}\right)^2-1=0$$



Pros

- Very concise
- Guaranteed validity
- Easy to test if points are on surface or inside
- Easy to intersect two shapes

Cons

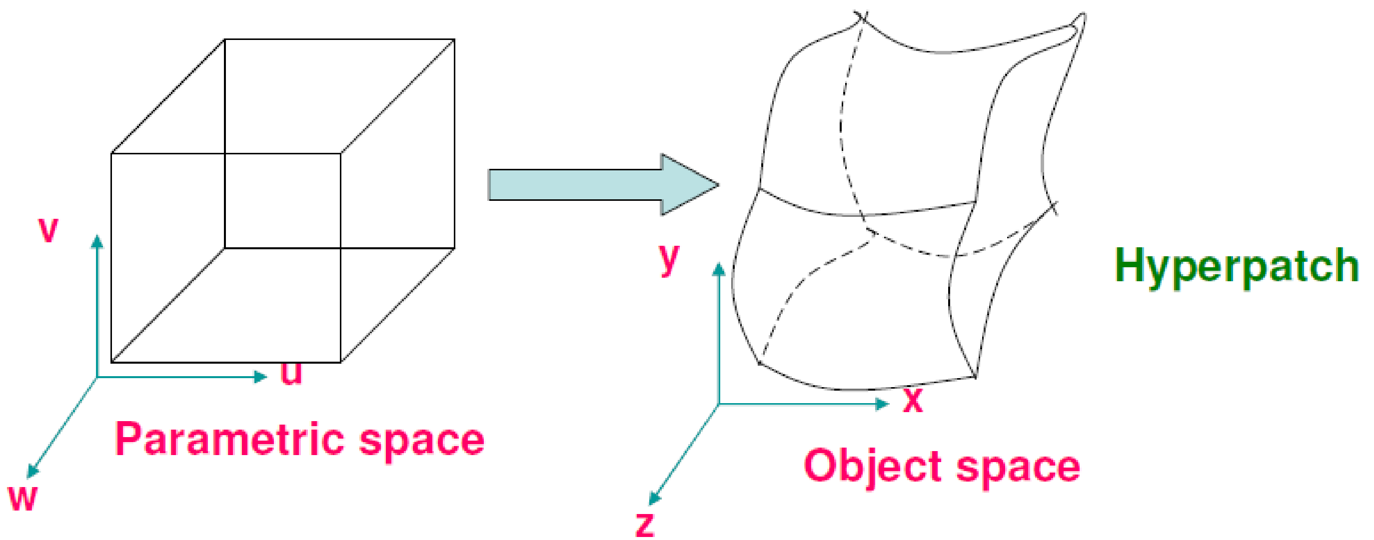
- Hard to describe complex shapes
- Hard to draw (interact with users)

Pictures from Google Image

CAD – Solid(3D) – Parametric representation

$$P(u,v,w)=[x \quad y \quad z]=[x(u,v,w) \quad y(u,v,w) \quad z(u,v,w)]$$

$$u_{\min} \le u \le u_{\max}; v_{\min} \le v \le v_{\max}; w_{\min} \le w \le w_{\max}$$



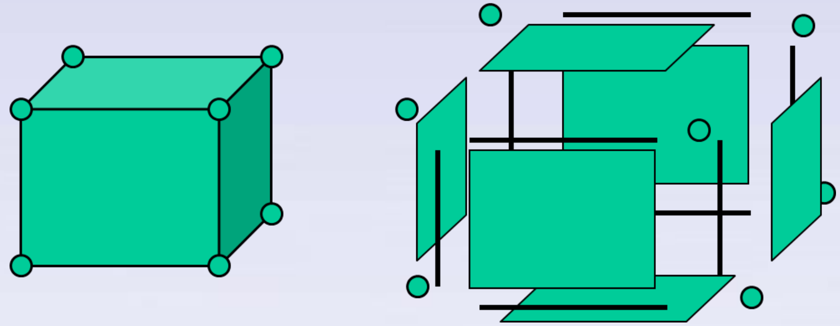
Pictures credit: Dr Shriram Hegde@IITD

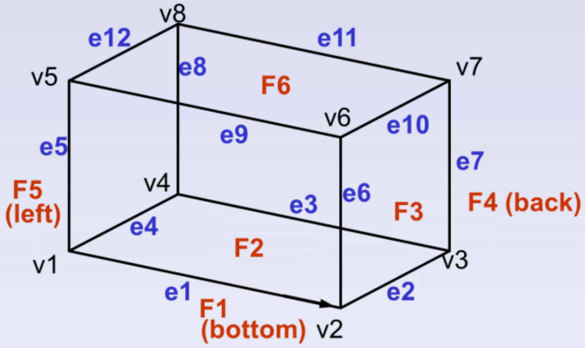
Geometry

CAD – Solid(3D) – Boundary representation (B-rep)

B-rep (Baumgart 1970) – Explicit representation of:

- Boundary of an object
- Connectivity among faces, edges, and vertices
- Geometric and topological information





Winged-Edge Tables

Edge	Vstart	Vend	fcw	fccw
e1	v1	v2	f1	f2
e2	v2	v3	f1	f3
e3	v3	v4	f1	f4
e4	v4	v1	f1	f5
e5	v1	v5	f2	f5

• • •

Pictures credit: Dr David Rosen@GaTech

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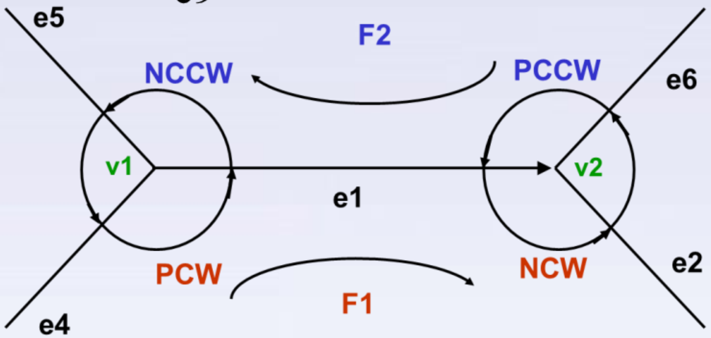
CAD – Solid(3D) – Boundary representation (B-rep)

Winged-Edge Tables

Vertex	Coords	First Edge
v1	x1 y1 z1	e1
v2	x2 y2 z2	e2
v3	. . . . .	e3
v4	. . . . .	e4
v5	. . . . .	e9
v6	. . . . .	e10
v7	. . . . .	e11
v8	. . . . .	e12

Face	First Edge
f1	e1
f2	e9
f3	e6
f4	e7
f5	e12
f6	e9

EDGE	NCW	PCW	NCCW	PCCW
e1	e2	e4	e5	e6
e2	e3	e1	e6	e7
e3	e4	e2	e7	e8
e4	e1	e3	e8	e5
e5	e9	e1	e4	e12
e6	e10	e2	e1	e9
e7	e11	e3	e2	e10
e8	e12	e4	e3	e11
e9	e6	e5	e12	e10
e10	e7	e6	e9	e11
e11	e8	e7	e10	e12
e12	e5	e8	e11	e9



Pros

- Explicit topological info
- Easy to render

Cons

- Hard to check validity
- Hard for Boolean operation

Pictures credit: Dr David Rosen@GaTech

CAD – Solid(3D) – Sweep representation

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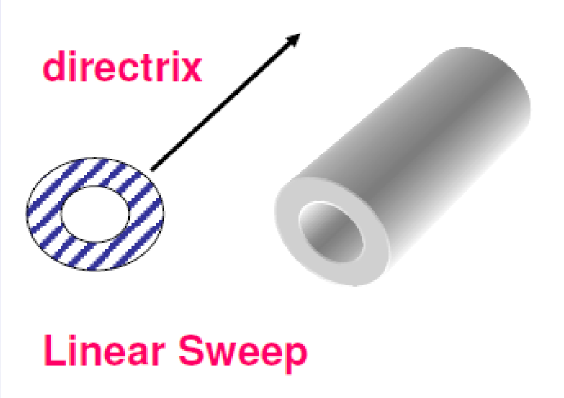
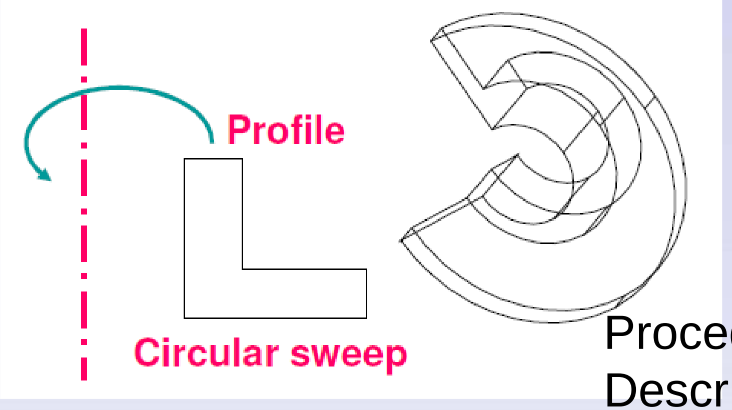




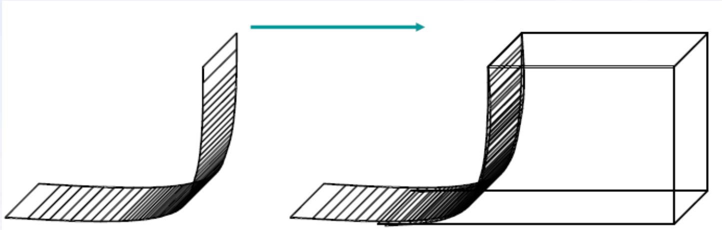
Procedural Modeling: Describe 3D models using algorithms

Pros

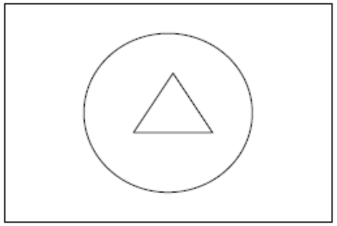
- Simple representation
- Good for uniform extruded objects or rotational geometry

Cons

- Limited domain
- No formal theory
- Validation schemes unknown



Invalid Sweep



Invalid 2-D profile for sweep, nested more than 1 level

Pictures credit: Dr Shriram Hegde@IITD

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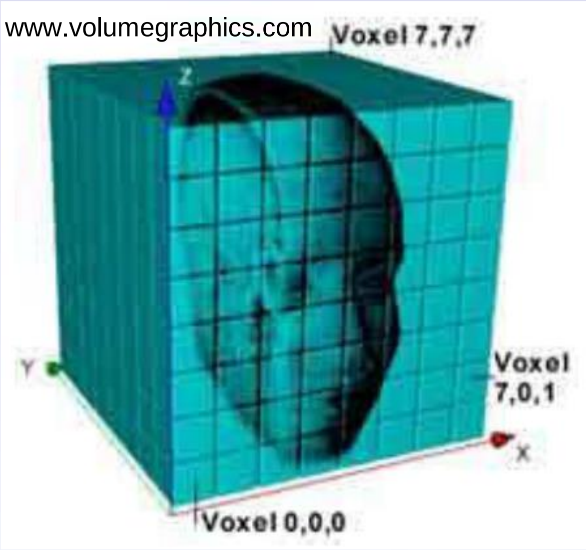
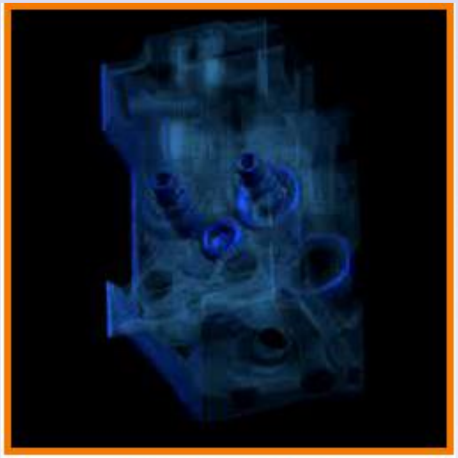
CAD – Solid(3D) – Spatial Occupancy Enumeration

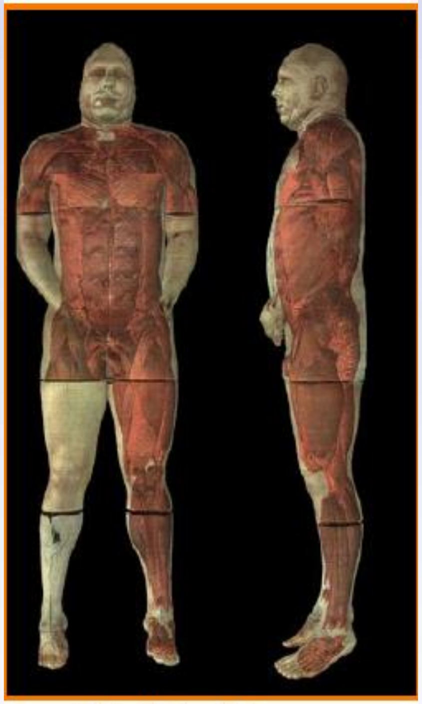
Partition space into a uniform grid

- Grid cells are called **voxels** (volume element, like pixels)





Engine Block
Stanford University



Visible Human
(National Library of Medicine)

Store properties of solid object with each voxel

- Occupancy
- Color
- Density
- Temperature
- etc.

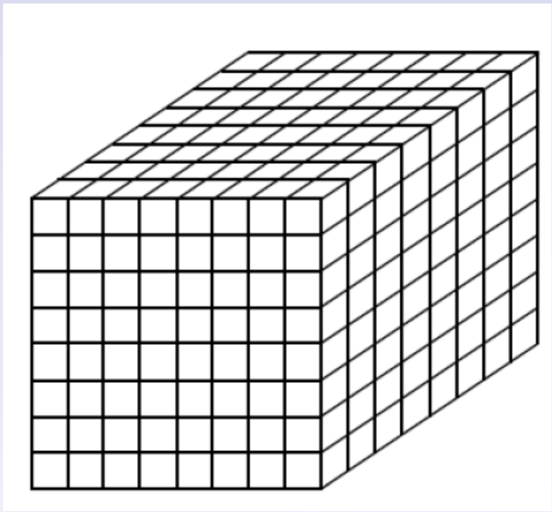
Slides credit: Dr Hong Qin@Stony Brook U

Geometry

CAD – Solid(3D) – Spatial Occupancy Enumeration

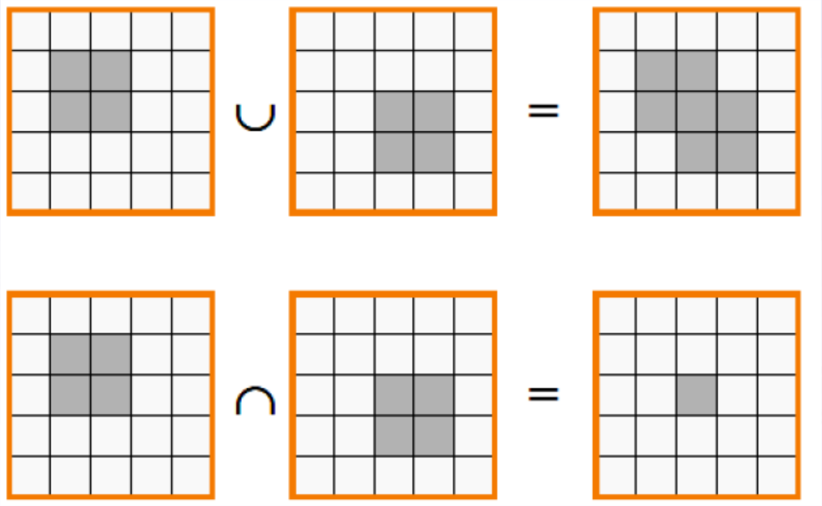
Voxel storage

- $O(n^3)$ storage fo nxnxn grid (1 Billion voxels for 1000x1000x1000)
- Processing just like image processing



Boolean operations

- Compare objects voxel by voxel
- Trivial



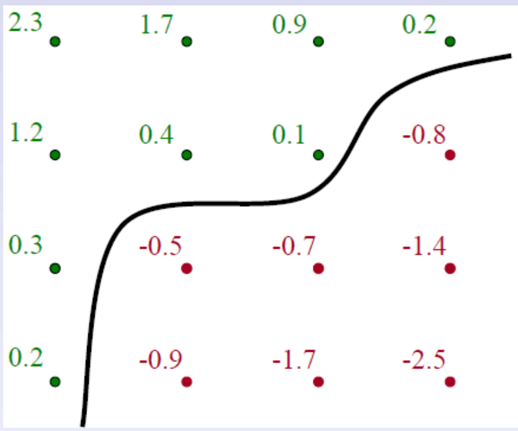
Slide credit - Dr. Wojciech Matusik@MIT

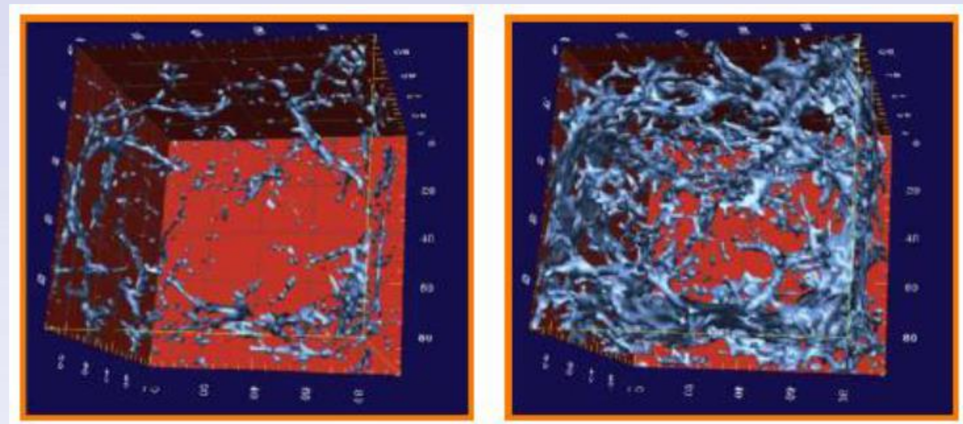
Geometry

CAD – Solid(3D) – Spatial Occupancy Enumeration

Voxel display

- Isosurface rendering: Render surfaces bounding volumetric regions of constant value (e.g., density)

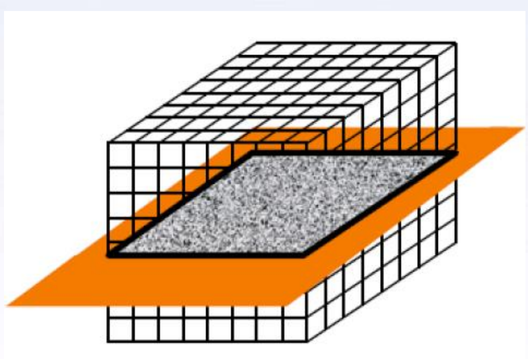




Isosurface visualization (Princeton University)

Slicing

- Draw 2D image from intersecting voxels with a plane



Visible Human (National Library of Medicine)

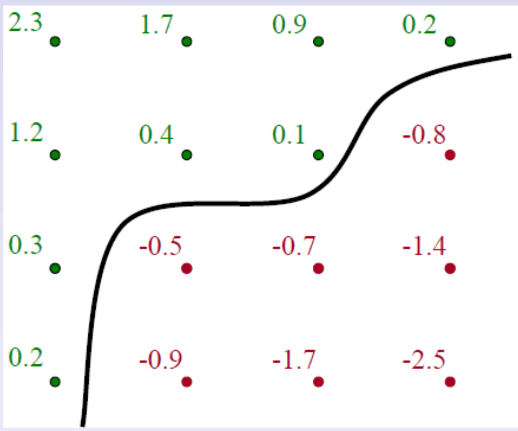
Slide credit - Dr. Wojciech Matusik@MIT

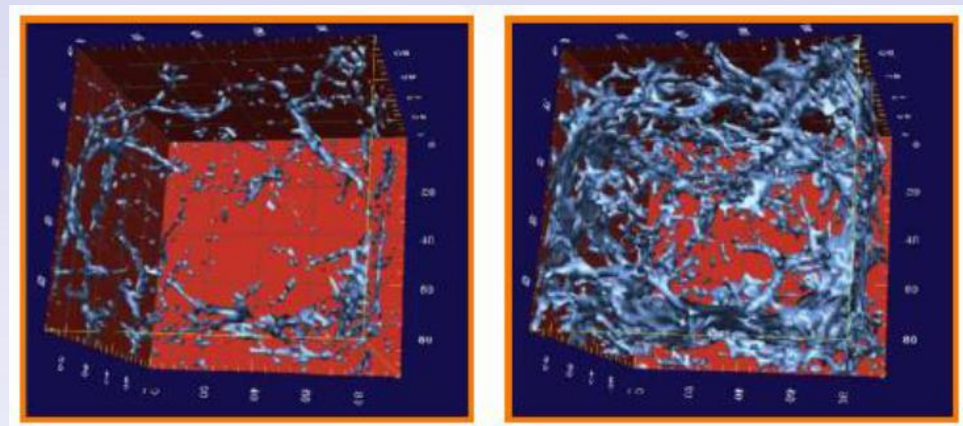
Geometry

CAD – Solid(3D) – Spatial Occupancy Enumeration

Voxel display

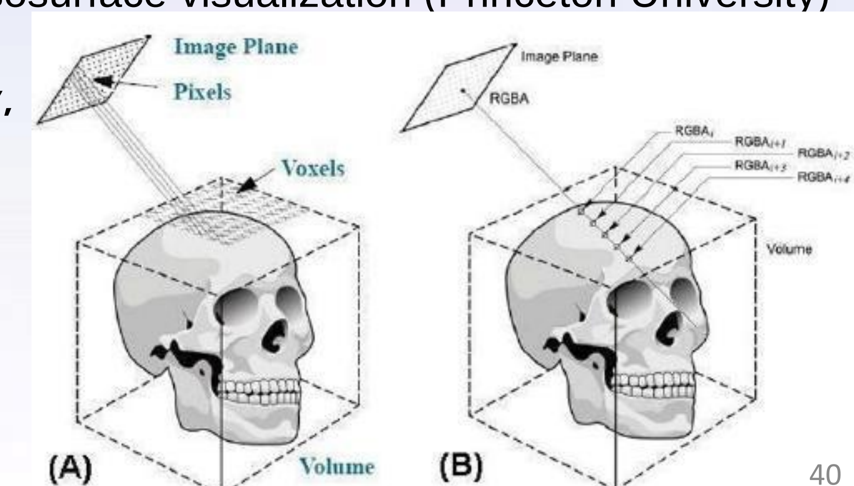
- **Isosurface rendering: Render surfaces bounding volumetric regions of constant value (e.g., density)**





Isosurface visualization (Princeton University)

- **Ray casting**
  - **Integrate RGB, opacity, etc. for rendering**



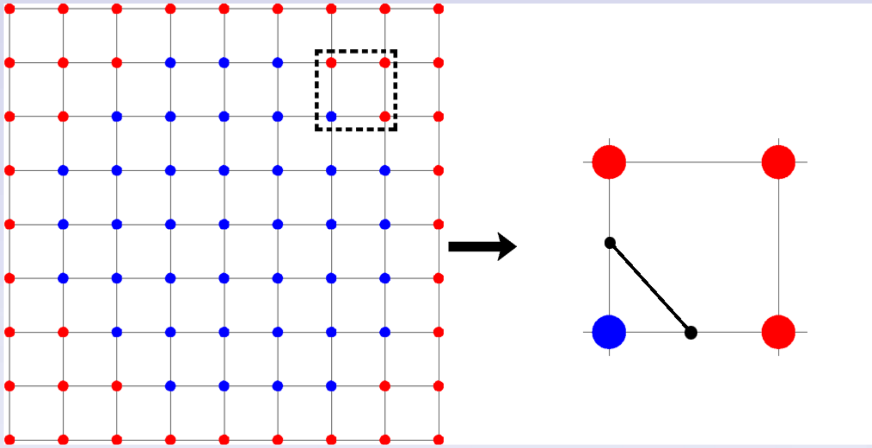
Guang Li, et al. Journal of Applied Clinical Medical Physics

Slide credit - Dr. Wojciech Matusik@MIT

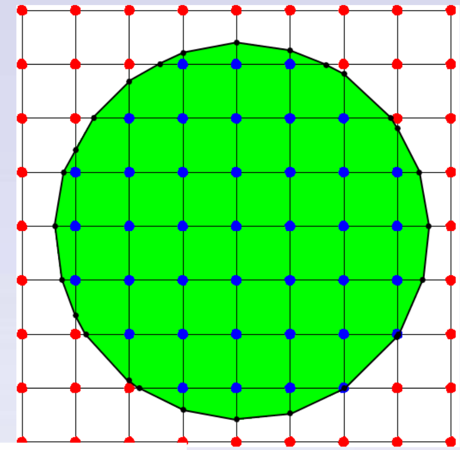
Geometry

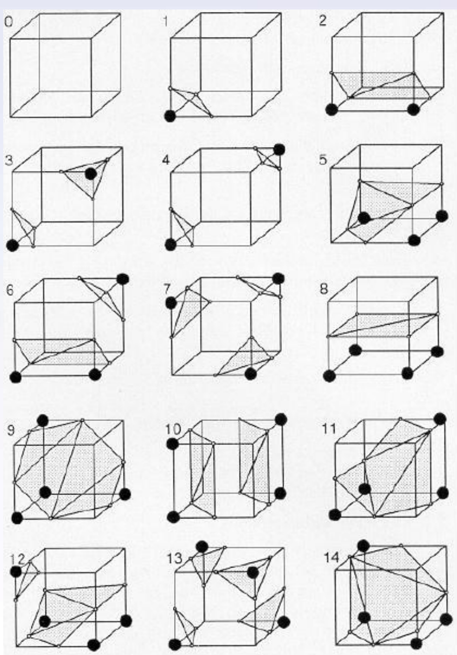
CAD – Solid(3D) – Spatial Occupancy Enumeration

Polygon generation

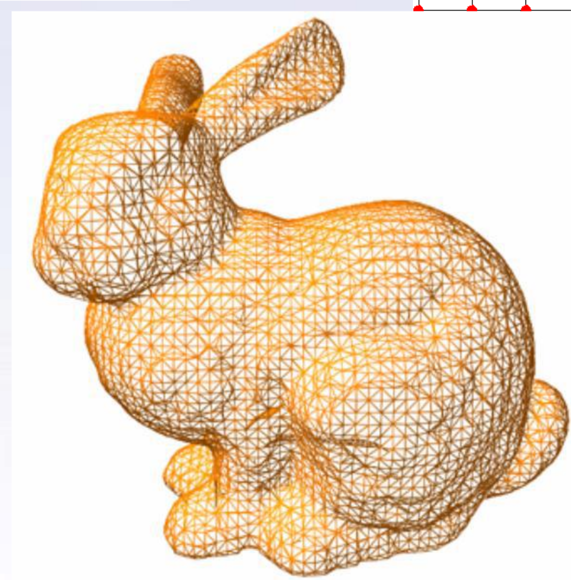


2D





3D



Slides credit: Dr Hong Qin@Stony Brook U

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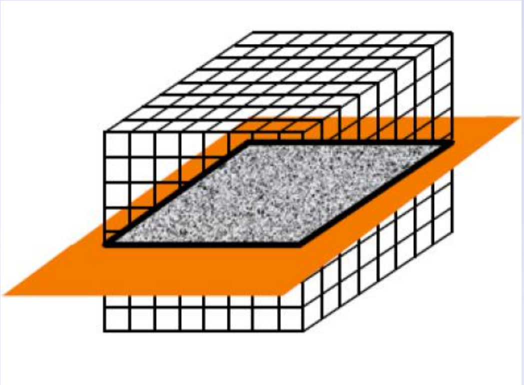
CAD – Solid(3D) – Spatial Occupancy Enumeration

Slicing

- **Draw 2D image from intersecting voxels with a plane**



Visible Human (National Library of Medicine)

Pros

- **Simple, intuitive, unambiguous**
- **Same complexity for all objects**
- **Natural acquisition for some apps.**
- **Trivial Boolean operations**

Cons

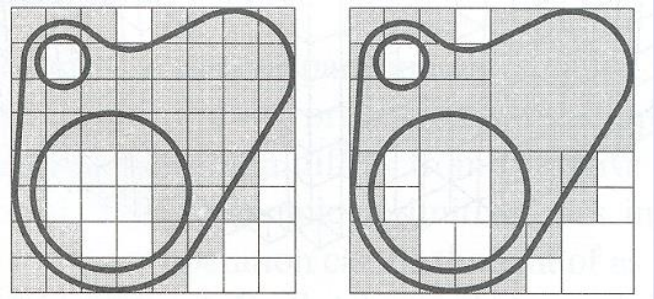
- **Approximate**
- **Not invariant for affine transformations**
- **Large storage requirements**
- **Expensive display**

Slide credit - Dr. Wojciech Matusik@MIT

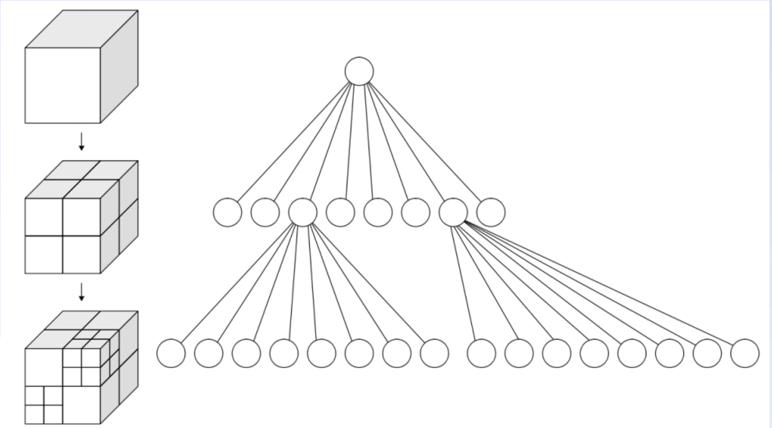
CAD – Solid(3D) – Spatial Occupancy Enumeration

Quadtrees & Octrees

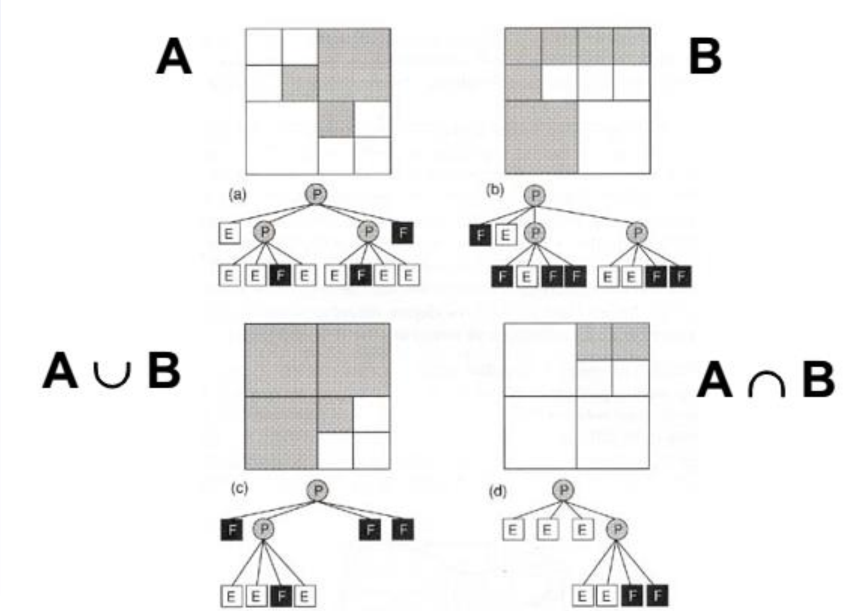
- Refine resolution of voxels hierarchically
- Encoded using a standard tree data structure



Uniform voxels   Quadtree



Octree in 3D



Boolean operations

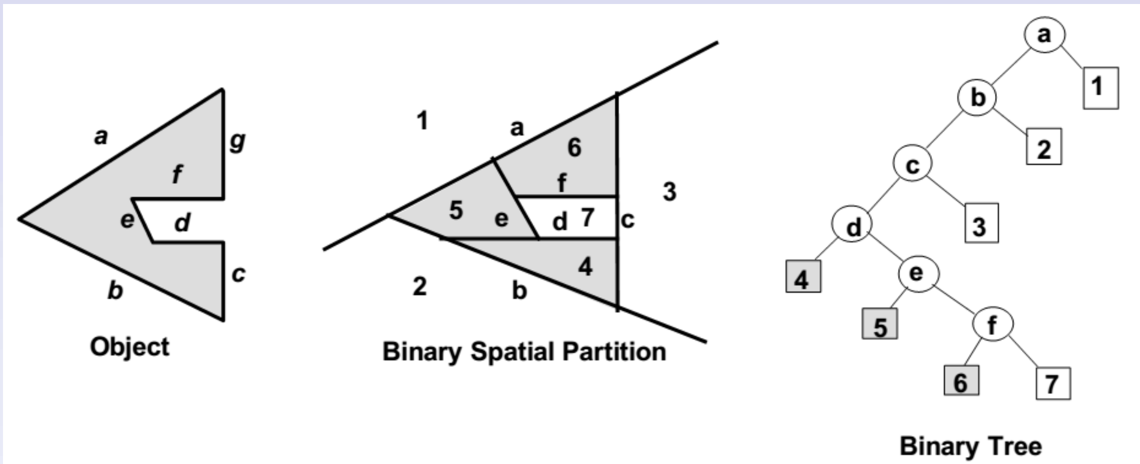
Slide credit - Dr. Wojciech Matusik@MIT

Geometry

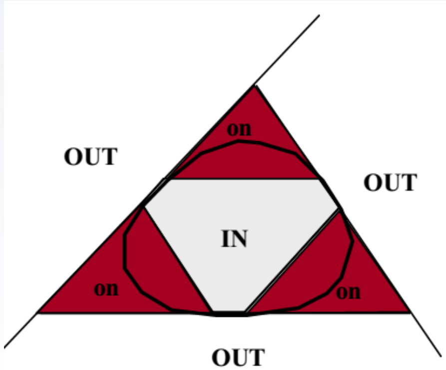
CAD – Solid(3D) – Spatial Occupancy Enumeration

Binary space partitions (BSPs)

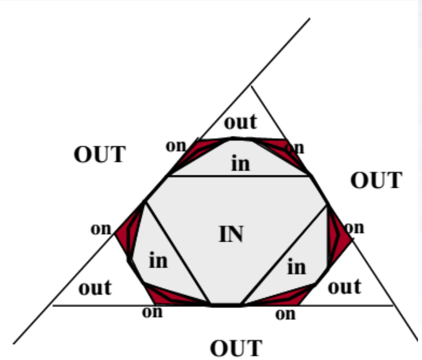
- Recursive partition of space by planes
- Make leaf cells as inside or outside object



- Regions decrease in size along tree depth and converge to the surface



1st level Approximation



2nd level Approximation

Slide credit Dr. Tom Funkhouser@Princeton

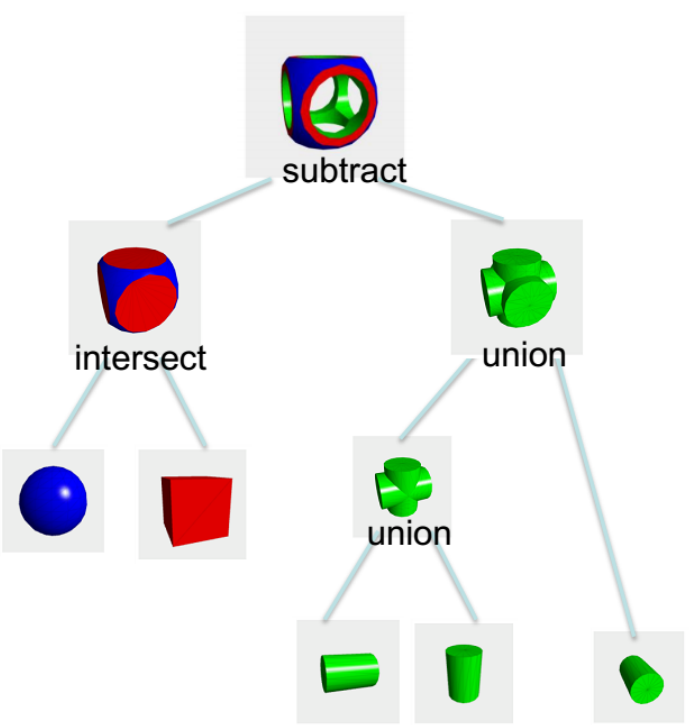
CAD – Solid(3D) – Constructive Solid Geometry (CSG)

CSG: 1974 by Ian Braid

- Build complex objects from simple parts using Boolean operations
- Intuitive
- Represent solid object as hierarchy of Boolean operations
- Boolean operations are not evaluated
- Objects are represented implicitly with a tree structure

Simple shapes

- Cuboids
- Cylinders
- Prisms
- Pyramids
- Spheres
- Cones
- Extrusions/Sweepings



Pictures courtesy Dr. Wojciech Matusik@MIT

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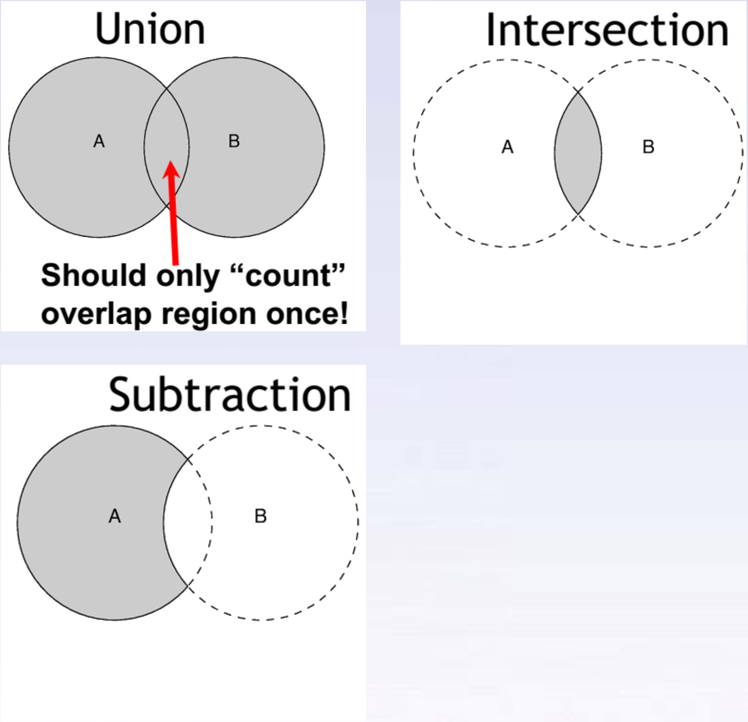
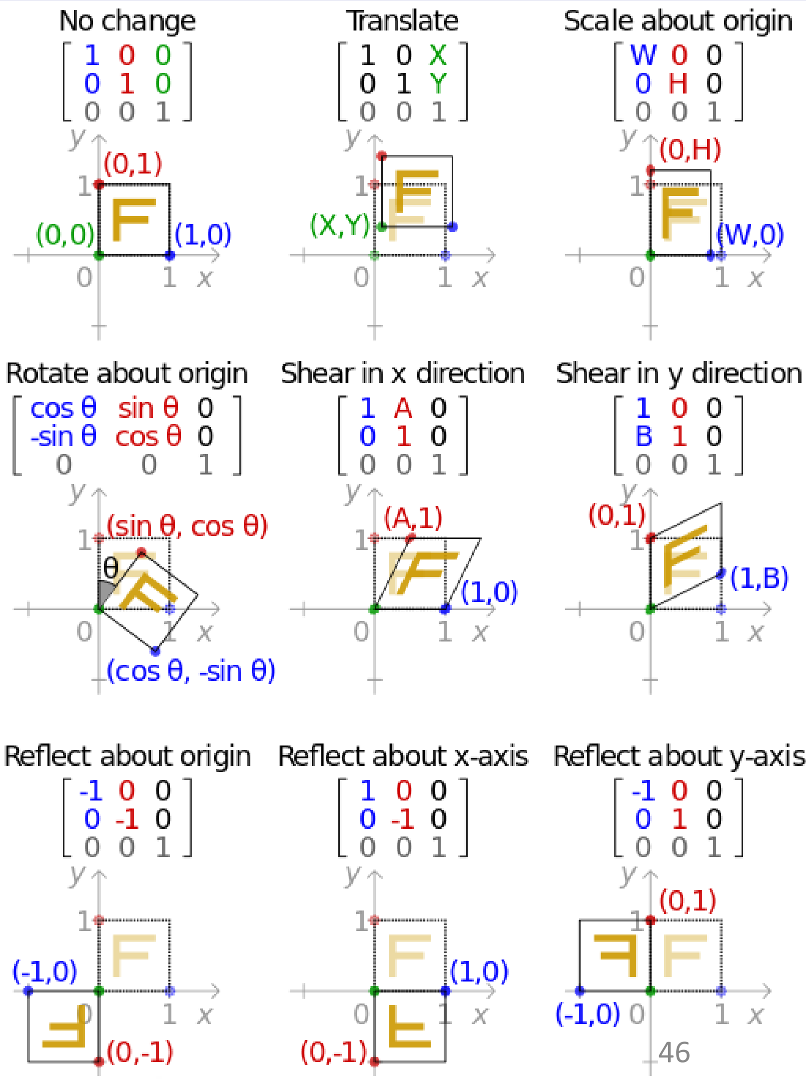
CAD – Solid(3D) – Constructive Solid Geometry (CSG)

Boolean operations

- Union
- Intersection
- Difference



Affine transformations



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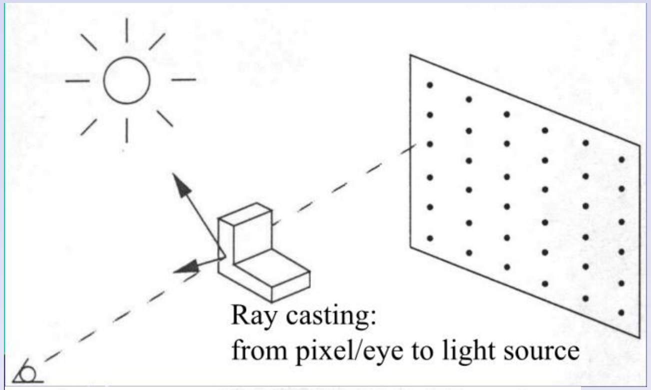
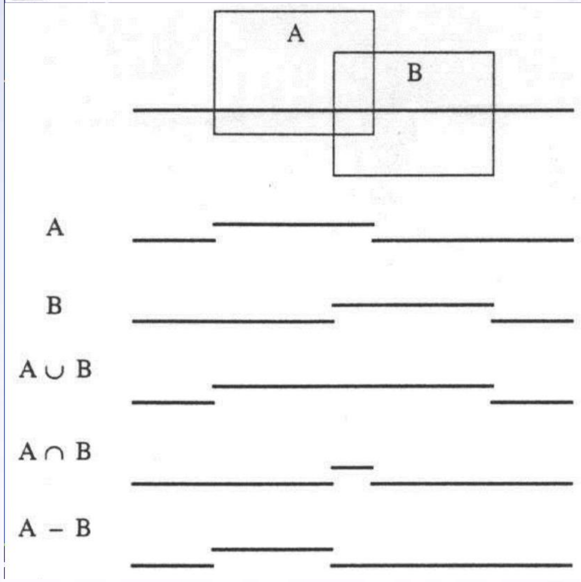
CAD – Solid(3D) – Constructive Solid Geometry (CSG)

Display & Analysis

- Ray Casting





Algorithm: Ray tracing

- INPUT: Assume that we have a ray R and a CSG tree T
- If T is a solid,
  - compute all intersections of R with T
  - return parameter values and normals
- If T is a transformation
  - apply inverse transformation to R and recursion
  - apply inverse transpose of transformation to normals
  - return parameter values
- Otherwise T is a Boolean operation
  - recursion on two children to obtain two sets of intervals
  - apply operation in T to intervals
  - return parameter values.
- OUTPUT: Display closest intersection points

Slides credit: Dr Hong Qin@Stony Brook U

Geometry

CAD – Solid(3D) – Constructive Solid Geometry (CSG)

Algorithm: Inside/Outside Test

- Given a point *p* and a tree *T*, determine if *p* is inside/outside the solid defined by *T*
- If *T* is a solid
  – Determine if *p* is inside *T* and return
- If *T* is a transformation
  – Apply the inverse transformation to *p* and recursion
- Otherwise *T* is a Boolean operation
  – Recursion to determine inside/outside of left/right children
  – If T is Union
  • If either child is inside, return inside, else outside
  – If T is Intersection
  • If both children are inside, return inside, else outside
  – If T is Subtraction
  • If *p* is inside left child and outside right child, return inside, else outside

Algorithm: Calculate volume

Put bounding box around object
• Pick n random points inside the box
– Determine if each point is inside/outside the CSG Tree
• Volume = #inside/n

Slides credit: Dr Hong Qin@Stony Brook U

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CAD – Solid(3D) – Summary

	Implicit/ Parametric	B-rep	Voxel	Octree	BSP	CSG
Accurate	Yes	Yes	No	No	Some	Some
Concise	Yes	Some	No	No	No	Yes
Affine invariant	Yes	Yes	No	No	Yes	Yes
Easy acquisition	No	No	Some	Some	No	Some
Guaranteed validity	Yes	No	Yes	Yes	Yes	No
Efficient Boolean operations	Yes	No	Yes	Yes	Yes	Yes
Efficient display	Yes	Yes	No	No	Yes	No
Expressive power	Very Limited	Good	Excellent	Excellent	Excellent	Excellent


THANK
YOU!